Equal-weighted strategy: Why it outperforms value-weighted strategies? Theory and evidence

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2 authors:

Rama K. Malladi
California State University, Dominguez Hills

Frank Fabozzi
Johns Hopkins University

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Rama Malladi
Kubera Investments LLC, Irvine, CA, USA
rmalladi@gmail.com

Frank J. Fabozzi
EDHEC Business School, Nice, France

Abstract

Recent academic papers and practitioner publications suggest that equal-weighted portfolios (or 1/N portfolios) appear to outperform various other portfolio strategies. In addition, as the equal-weighted portfolio does not rely on expected average returns, it is therefore assumed to be more robust compared to other price-weighted or value-weighted strategies. In this paper we provide a theoretical framework to the equal-weighed versus value-weighted equity portfolio model, and demonstrate using simulation as well as real-world data from 1926 to 2014 that an equal-weighted strategy indeed outperforms value-weighted strategies. Moreover, we demonstrate that a significant portion of the excess return is attributable to portfolio rebalancing. Finally, we show that because of equal-weighting the excess returns are higher than the higher costs incurred due to higher portfolio turnover. Therefore, even after accounting for higher portfolio turnover costs, equal-weighting makes economic sense.

JEL classification: G11

Keywords: Equal-weighted, cap-weighted, 1/N, asset allocation, portfolio optimization

1 Introduction

Since the introduction of the S&P 500 index in 1957, most indices have been weighted by market capitalization (or value-weighted, abbreviated as VW). By the end of 2014, S&P Dow Jones Indices estimates that over \$7.8 trillion was benchmarked to the S&P 500 alone, with indexed assets making up \$2.2 trillion of this total.[1] The theoretical foundation for VW indices as a benchmark for investors is provided by the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), and the Efficient Market Hypothesis (EMH) of Fama (1970). According to the CAPM model, the expected return implicit in the price of a stock should be commensurate with the risk of that stock. Based on the CAPM and EMH theory, the most efficient portfolio would be the entire market and a broad VW index would represent the optimal mean-variance efficient investment. However, following the critique by Roll (1977), there has been considerable debate (examples of which are available in Gruber and Ross (1978) and Gibbons (1982)) as to how efficient the market portfolio is in practice. Thus, there are countless different strategies[2] as documented by Arnott et al. (2005) to beat the market. This has led to indices created based on alternative factors that measure different strategies, sometimes referred to as Smart Beta strategies, as explained by Amenc et al. (2011b) and Amenc and Goltz (2013). Investors have been attracted by the performance of these indices compared to traditional cap-weighted indices. Some of the popular alternative indices are FTSE EDHEC-Risk Efficient Index, Intech's Diversity-Weighted Index, Research Affiliates Fundamental Index, QS Investors' Diversification Based Investing (DBI), and TOBAM's Maximum Diversification Index.

Recent studies by DeMiguel et al. (2009) and Plyakha et al. (2015) suggest that equal-weighted portfolios (also known as 1/N, abbreviated as EWP) appear to outperform 14 different portfolio weighting strategies. The out-of-sample performance of an EWP of stocks is significantly better than that of a value-weighted portfolio (abbreviated as VWP), and no worse than that of portfolios from a number of optimal portfolio selection models. Plyakha et al. (2014) report that for the 14 models that they studied using seven empirical datasets, none were consistently better than the EWP in terms of Sharpe ratio, certainty-equivalent return, or turnover. Because the EWP

[1] http://www.spindices.com/documents/index-policies/spdji-indexed-assets-survey-2014.pdf

[2] http://faculty-research.edhec.com/_medias/fichier/edhec-position-paper-smart-beta-2-0_1378195044229-pdf

does not rely on expected average returns, it is therefore assumed to be more robust compared to other price-weighted or VW strategies. Moreover an EW strategy minimizes the risk of a portfolio deviating from the investor's target allocation objectives.

In this paper our objective is not to support one of the several alternative beta strategies in the market, as they all seem to outperform VW indices according to Arnott et al. (2005), Chow et al. (2011), and Amenc et al. (2011a). Rather, we focus on the nature and source of EWP returns. Given the positive excess returns of EWPs relative to VWPs in the equity asset class, two natural questions arise: (1) Can this excess return be realized under various market scenarios and time horizons? and (2) What is the source of these positive excess returns? Plyakha et al. (2015) identified rebalancing as the main source of excess returns by using two experiments with simulated data. We develop a model to explain the source of excess returns, and test it with both simulated and real-world data. In this paper we provide a theoretical framework for the EWP model, and demonstrate using simulation and historical data that an EW strategy indeed outperforms VW strategies. Then we show that rebalancing is a key driver behind the positive excess retun of the EWPs

To undertake our analysis, a two-period equity portfolio model with two assets is developed. This model can be extended to multiple periods as well as multiple assets. This equity portfolio model proves that after portfolio rebalancing, if smaller-cap stocks outperform larger-cap stocks, then the EWP will produce higher returns than the VWP. Given that smaller-cap stocks are riskier than larger-cap stocks, it is quite natural to expect a higher return from smaller-cap stocks. So it is plausible for an EWP to outperfom a VWP. We validate the model with five tests to see if EWPs produce higher returns and Sharpe ratios than VWPs.

The flow of this paper is as follows. Section 2 describes the portfolio model for VWP and EWP. In Section 3 we describe the data used. The results are presented and discussed in Section 4, followed by our conclusions in Section 5.

2 Portfolio Model

In this section, an equity portfolio is built with two (n=2) stocks A and B. Either A or B can be a large cap stock. At t=0, an investor's wealth is invested in either a VW or EW portfolio. At

t=1, single-period returns of value-weighted portfolio (denoted as V) and equal-weighted portfolio (denoted as E) are computed. The main difference between the two portfolios (V and E) is that equal-weighted portfolio has to be rebalanced after t=1, whereas the value-weighted portfolio does not require rebalancing. At t=2, the returns denoted by R_{2V} and R_{2E}, the standard deviation of returns denoted by σ_{2V} and σ_{2E}, and Sharpe ratios denoted by S_{2V} and S_{2E} are computed. In the end, the following five metrics are computed:

1. Excess return of EWP over VWP: $R_{2E} - R_{2V}$
2. Excess risk of EWP over VWP: $\sigma_{2E} - \sigma_{2V}$
3. Excess Sharpe ratio of EWP over VWP: $S_{2E} - S_{2V}$
4. Decomposed excess return due to rebalancing effect
5. Decomposed excess return due to size effect.

2.1 Value-weighted portfolio return

Let M_0 denote the investable wealth (or portfolio value) at t=0 and then

Marketcap of firm A at t=0 is: $\mathrm{V}_{A0} = P_{A0}Q_A$, where P_{A0} is price of A, Q_A is number of shares outstanding

Marketcap of firm B at t=0 is: $\mathrm{V}_{B0} = P_{B0}Q_B$, where P_{B0} is price of B, Q_B is number of shares outstanding

Portfolio value invested in firm A, at t=0, according to value - weight is: $M_{A0V} = \dfrac{M_0 P_{A0} Q_A}{P_{A0}Q_A + P_{B0}Q_B}$

Portfolio value invested in firm B, at t=0, according to value - weight is: $M_{B0V} = \dfrac{M_0 P_{B0} Q_B}{P_{A0}Q_A + P_{B0}Q_B}$

where $M_0 = M_{A0V} + M_{B0V}$

Number of firm A stocks in portfolio at t=0: $N_{A0V} = \dfrac{M_{A0V}}{P_{A0}} = \dfrac{M_0 Q_A}{P_{A0}Q_A + P_{B0}Q_B}$

Number of firm B stocks in portfolio at t=0: $N_{B0V} = \dfrac{M_{B0V}}{P_{B0}} = \dfrac{M_0 Q_B}{P_{A0}Q_A + P_{B0}Q_B}$

Similarly at t=1: $\mathrm{V}_{A1} = P_{A1}Q_A$, $\mathrm{V}_{B1} = P_{B1}Q_B$, $M_{A1V} = N_{A0V}P_{A1}$, $M_{B1V} = N_{B0V}P_{B1}$

Value-weighted portfolio return between t=0 and 1 = $R_{1V} = \dfrac{M_{A1V} + M_{B1V}}{M_{A0V} + M_{B0V}} - 1$

$$\Rightarrow R_{1V} = \frac{Q_A\left(P_{\mathrm{A1}} - P_{\mathrm{A0}}\right) + Q_B\left(P_{\mathrm{B1}} - P_{\mathrm{B0}}\right)}{Q_A P_{\mathrm{A0}} + Q_B P_{\mathrm{B0}}} \tag{1}$$

Since the VWP is not rebalanced after t=1, $N_{A1V} = N_{A0V}, N_{B1V} = N_{B0V}$

Similarly at t=2: $\mathrm{V}_{A2} = P_{A2}Q_A$, $\mathrm{V}_{B2} = P_{B2}Q_B$, $M_{A2V} = N_{A1V}P_{A2}$, $M_{B2V} = N_{B1V}P_{B2}$

Value-weighted portfolio return between t=1 and 2 = $R_{2V} = \dfrac{M_{A2V} + M_{B2V}}{M_{A1V} + M_{B1V}} - 1$

$$\Rightarrow R_{2V} = \frac{Q_A\left(P_{\mathrm{A2}} - P_{\mathrm{A1}}\right) + Q_B\left(P_{\mathrm{B2}} - P_{\mathrm{B1}}\right)}{Q_A P_{\mathrm{A1}} + Q_B P_{\mathrm{B1}}} \tag{2}$$

2.2 Equal-weighted portfolio return

Again, letting M_0 denote the investable wealth (or portfolio value) at t=0 and then

Marketcap of firm A at t=0 is: $\mathrm{V}_{A0} = P_{A0}Q_A$, where P_{A0} is price of A, Q_A is number of shares outstanding

Marketcap of firm B at t=0 is: $\mathrm{V}_{B0} = P_{B0}Q_B$, where P_{B0} is price of B, Q_B is number of shares outstanding

Given that n = 2, 50% of the portfolio will be invested in A and B each.

$\Rightarrow M_{A0E} = M_{B0E} = \dfrac{M_0}{2}$, $N_{A0E} = \dfrac{M_0}{2P_{A0}}$, $N_{B0E} = \dfrac{M_0}{2P_{B0}}$, $M_{A1E} = N_{A0E}P_{A1}$, $M_{B1E} = N_{B0E}P_{B1}$

Equal-weighted portfolio return between t=0 and 1 = $R_{1E} = \dfrac{M_{A1E} + M_{B1E}}{M_{A0E} + M_{B0E}} - 1$

$$\Rightarrow R_{1E} = \left(\frac{P_{A1}}{2P_{A0}} + \frac{P_{B1}}{2P_{B0}}\right) - 1 \tag{3}$$

Unlike the value-weighted portfolio, the equal-weighted portfolio weights have to be rebalanced after t=1. Given the portfolio value of $M_{A1E} + M_{B1E}$ at t=1 before rebalancing, the portfolio value of A and B after rebalancing will be $M_{A1ER} = M_{B1ER} = \dfrac{1}{2}\left(M_{A1E} + M_{B1E}\right)$.

As a result, $N_{A1ER} = \dfrac{M_{A1ER}}{P_{A1}}$, and $N_{B1ER} = \dfrac{M_{B1ER}}{P_{B1}}$.

So at t=2: $\mathrm{V}_{A2} = P_{A2}Q_A$, $\mathrm{V}_{B2} = P_{B2}Q_B$, $M_{A2E} = N_{A1ER}P_{A2}$, $M_{B2E} = N_{B1ER}P_{B2}$

Equal-weighted portfolio return between t=1 and 2 = $R_{2E} = \dfrac{M_{A2E} + M_{B2E}}{M_{A1E} + M_{B1E}} - 1$

$$\Rightarrow R_{2E} = \left(\frac{P_{A2}}{2P_{A1}} + \frac{P_{B2}}{2P_{B1}}\right) - 1 \tag{4}$$

2.3 Excess return for the portfolio

We define excess return, $R_{2E} - R_{2V}$, as the EWP return minus the VWP return at t=2.

$$R_{2E} - R_{2V} = \left(\frac{P_{A2}}{2P_{\mathrm{A1}}} + \frac{P_{B2}}{2P_{\mathrm{B1}}}\right) - 1 - \frac{Q_A\left(P_{\mathrm{A2}} - P_{\mathrm{A1}}\right) + Q_B\left(P_{\mathrm{B2}} - P_{\mathrm{B1}}\right)}{Q_A P_{\mathrm{A1}} + Q_B P_{\mathrm{B1}}}$$

$$\Rightarrow R_{2E} - R_{2V} = \frac{Q_A P_{\mathrm{A1}}^2 P_{\mathrm{B2}} + P_{\mathrm{A2}} Q_B P_{\mathrm{B1}}^2 - P_{\mathrm{A1}} P_{\mathrm{B1}}\left(Q_A P_{\mathrm{A2}} + Q_B P_{\mathrm{B2}}\right)}{2P_{\mathrm{A1}} P_{\mathrm{B1}}\left(Q_A P_{\mathrm{A1}} + Q_B P_{\mathrm{B1}}\right)} \tag{5}$$

$$\Rightarrow R_{2E} - R_{2V} = \frac{\left(P_{\mathrm{A1}} P_{B2} - P_{\mathrm{B1}} P_{A2}\right)\left(P_{\mathrm{A1}} Q_A - P_{\mathrm{B1}} Q_B\right)}{2P_{\mathrm{A1}} P_{\mathrm{B1}}\left(Q_A P_{\mathrm{A1}} + Q_B P_{\mathrm{B1}}\right)} \tag{6}$$

Based on equation (6), at t=2, the EWP will produce a higher return than a VWP if and only if $R_{2E} - R_{2V} > 0$. This is possible under the two scenarios discussed next.

Scenario (a): $P_{\mathrm{A1}} P_{B2} - P_{\mathrm{B1}} P_{A2} > 0 \Rightarrow P_{B2}/P_{\mathrm{B1}} > P_{A2}/P_{A1} \Rightarrow$ B has a higher return than A.

and $P_{\mathrm{A1}} Q_A - P_{\mathrm{B1}} Q_B > 0 \Rightarrow P_{\mathrm{A1}} Q_A > P_{\mathrm{B1}} Q_B \Rightarrow$ B has a smaller cap compared to A.

Together the above two conditions imply that if the return of the small cap stock is greater than that of the large cap stock, then the excess return will be positive.

Scenario (b): $P_{\mathrm{A1}} P_{B2} - P_{\mathrm{B1}} P_{A2} < 0 \Rightarrow P_{B2}/P_{\mathrm{B1}} < P_{A2}/P_{A1} \Rightarrow$ A has higher return than B.

and $P_{\mathrm{A1}} Q_A - P_{\mathrm{B1}} Q_B < 0 \Rightarrow P_{\mathrm{A1}} Q_A < P_{\mathrm{B1}} Q_B \Rightarrow$ A has a smaller cap compared to B.

As in scenario (a), the above two conditions imply that if the return of the small cap stock exceeds that of the large cap stock, then the excess return will be positive.

In summary, in both scenarios (a) and (b) after rebalancing the portfolio at t=1, if the smaller-cap stock outperforms the larger-cap stock, then the EWP will produce higher returns than a VWP. Given that the smaller-cap stocks are riskier than the larger-cap stocks, it is quite natural to expect a higher return from the smaller-cap stock in the long-run, thus it follows that because of portfolio rebalancing, an EWP will produce higher returns than a VWP. The same logic can be extended to the multi-period and multi-asset cases.

2.4 Excess risk (Standard Deviation)

In this section, excess risk is defined as the difference between the EWP standard deviation and the VWP standard deviation, denoted by σ_E-σ_V. If at t=2, the standard deviation for the large cap (L) and the small cap (S) returns are denoted by σ_L and σ_S, and the correlation between those returns is denoted by ρ, then the portfolio variance

$$\sigma_P^2 = W_L^2\sigma_L^2 + W_S^2\sigma_S^2 + 2W_LW_S\sigma_L\sigma_S\rho$$

For EWP, $W_L = W_S = 0.5$. Letting σ_S - σ_L = d, then

$$\sigma_E^2 = 0.25\left(\sigma_L^2 + \sigma_S^2\right) + 0.5\sigma_L\sigma_S\rho = 0.25\left(\sigma_L^2 + (\sigma_L + d)^2\right) + 0.5\sigma_L\rho\left(\sigma_L + d\right) \quad (7)$$

For VWP, $W_L > 0.5$. Letting $W_L = 0.5 + e$, where $0 < e < 0.5 \Rightarrow W_S = 0.5 - e$, then

$$\sigma_V^2 = (0.5+e)^2\sigma_L^2 + (0.5-e)^2(\sigma_L + d)^2 + 2\,(0.5+e)\,(0.5-e)\,\sigma_L\,(\sigma_L + d)\,\rho \quad (8)$$

$$\Rightarrow \sigma_E^2 - \sigma_V^2 = 0.5d^2(1-e)e + de(1 + e(\rho-1))\sigma_L + e^2(\rho-1)\sigma_L^2 \quad (9)$$

The EWP will have a higher variance than the VWP if $\sigma_E^2 - \sigma_V^2 = (\sigma_E - \sigma_V)(\sigma_E + \sigma_V) > 0$. Since $(\sigma_E + \sigma_V) \geqslant 0$ always, the excess risk will be positive when σ_E-$\sigma_V > 0$. Excess risk is therefore a function of $f(d, e, \rho, \sigma_L)$. EWP will have a higher variance depending on the value of d (difference in standard deviation of small cap and large cap), e (large cap weight minus equal weight), and ρ (correlation between large cap and small cap). Given that equation (9) is a function of d, e, ρ, and σ_L, when excess risk is positive, the following parameter values are realized.

$$d > \frac{\sigma_L\left[1 + e(\rho-1) \pm \sqrt{1 + e^2(\rho^2-1)}\right]}{e-1} \quad (10)$$

$$e > \frac{d^2 + 2d\sigma_L}{d^2 + 2\sigma_L(1-\rho)(d + \sigma_L)} \quad (11)$$

$$\sigma_L > \frac{-d\left[1 + e(\rho-1) \pm \sqrt{1 + e^2(\rho^2-1)}\right]}{2e(\rho-1)} \quad (12)$$

$$\rho > \frac{(e+1){\sigma_L}^2 + (e-1)(d+\sigma_L)^2}{2\sigma_L(d+\sigma_L)e} \tag{13}$$

2.5 Excess Sharpe Ratio

The excess Sharpe ratio is defined as EWP Sharpe ratio minus VWP Sharpe ratio, denoted by $S_E - S_V$. If at t=2, the realized annual returns of small cap and large cap are R_S, R_L respectively, annual risk-free rate is R_F, standard deviations of large cap and small cap returns are σ_L, σ_S, and the correlation between large cap and small cap returns is ρ,

$$\text{EWP return, } R_E = 0.5(R_L + R_S)$$

$$\text{VWP return, } R_V = R_L(0.5+e) + R_S(0.5-e)$$

Then the Sharpe ratios of EWP and VWPs are

$$S_E = \frac{0.5(R_L + R_S) - R_F}{\sqrt{0.25{\sigma_L}^2 + 0.5\rho\sigma_L\left(d+\sigma_L\right) + 0.25(d+\sigma_L)^2}} \tag{14}$$

$$S_V = \frac{R_L(0.5+e) + R_S(0.5-e) - R_F}{\sqrt{(0.5+e)^2{\sigma_L}^2 + 2\left(0.5-e\right)\left(0.5+e\right)\rho\sigma_L\left(d+\sigma_L\right) + (0.5-e)^2(d+\sigma_L)^2}} \tag{15}$$

$$S_{E-V} = \text{Excess Sharpe Ratio} = S_E - S_V \tag{16}$$

2.6 Rebalancing and Size effect

If the EWP has two stocks consisting of a large cap (A) and small cap (B), and produces a positive excess return, a natural question arises about the source of these excess returns. EWP and VWPs differ only in terms of weight of the small cap stocks and rebalancing. So the excess return can be attributed to one of these two. In this section, the excess return is decomposed into excess return attributable to the rebalancing effect and excess return attributable to the size (small-cap) effect.

In the Section 2.2, the EWP return is computed using equation (4), when the portfolio is rebalanced after t=1. If the EWP is not rebalanced after t=1, then the number of stocks in the non-rebalanced EWP will continue to be N_{A1E}, and N_{B1E}, instead of N_{A1ER}, and N_{B1ER} as in

Section 2.2. So without rebalancing, the portfolio value of the non-rebalanced EWP at t=2 will be $M_{2E\_NR} = M_{A2E\_NR} + M_{B2E\_NR}$, where $M_{A2E\_NR} = N_{A1E}P_{A2},\ M_{B2E\_NR} = N_{B1E}P_{B2}$.

$$\text{The non-rebalanced EWP return between } t\text{=1 and 2} = R_{2E\_NR} = \frac{M_{2E\_NR}}{M_{1E}} - 1$$

$$\Rightarrow R_{2E\_NR} = \frac{M_{A2E\_NR} + M_{B2E\_NR} - M_{1E}}{M_{1E}} = \frac{P_{\mathrm{B}0}(P_{A2} - P_{A1}) + P_{\mathrm{A}0}(P_{B2} - P_{B1})}{P_{\mathrm{A}1}P_{\mathrm{B}0} + P_{A0}P_{B1}} \tag{17}$$

The excess return, $R_{2E} - R_{2V}$, can be decomposed into excess return due to rebalancing effect and excess return due to size (small-cap) effect as shown below.

$$\text{Excess Return} = R_{2E} - R_{2V} = \underbrace{(R_{2E} - R_{2E\_NR})}_{Rebalancing\, effect} + \underbrace{(R_{2E\_NR} - R_{2V})}_{Size\, effect} \tag{18}$$

$$\text{Rebalancing Effect} = R_{2E} - R_{2E\_NR} = \frac{P_{A0}P_{A2}P_{B1}^2 + P_{B0}P_{B2}P_{A1}^2 - P_{A1}P_{B1}\left(P_{A2}P_{B0} + P_{A0}P_{B2}\right)}{2P_{A1}P_{B1}\left(P_{A1}P_{B0} + P_{A0}P_{B1}\right)} \tag{19}$$

$$\text{Size Effect} = R_{2E\_NR} - R_{2V} = \frac{\left(P_{A2}P_{B1} - P_{\mathrm{A}1}P_{B2}\right)\left(P_{\mathrm{B}0}Q_B - P_{\mathrm{A}0}Q_A\right)}{\left(P_{A1}P_{B0} + P_{A0}P_{B1}\right)\left(Q_AP_{\mathrm{A}1} + Q_BP_{\mathrm{B}1}\right)} \tag{20}$$

As shown in equation (19), the rebalancing effect does not depend on the market cap of stocks in the portfolio, and dependents solely on price movements. The return due to rebalancing is positive, i.e., $R_{2E} - R_{2E\_NR} > 0$, when $P_{A0}P_{A2}P_{B1}^2 + P_{B0}P_{B2}P_{A1}^2 > P_{A1}P_{B1}\left(P_{A2}P_{B0} + P_{A0}P_{B2}\right)$. By further simplification, it can be shown that $R_{2E} - R_{2E\_NR} > 0$ when $R_{B1} > R_{A1}$. In other words when the small cap return is more than the large cap return at t=1, the rebalancing effect for the EWP is going to be positive. Given that small cap stocks are riskier than the large cap stocks, it is quite natural to expect a higher return from the small cap stock in the long-run, thus it follows that because of portfolio rebalancing, an EWP will produce higher returns than a VWP.

As shown in equation (20), the size effect depends on both price fluctuations and the size of stocks. The return due to size effect is positive, i.e. $R_{2E\_NR} - R_{2V} > 0$, when $P_{A2}P_{B1} < P_{\mathrm{A}1}P_{B2}$ and $P_{\mathrm{B}0}Q_B < P_{\mathrm{A}0}Q_A \Rightarrow \frac{P_{A2}}{P_{\mathrm{A}1}} < \frac{P_{B2}}{P_{B1}}$ and $P_{\mathrm{B}0}Q_B < P_{\mathrm{A}0}Q_A$. This implies that size effect does not depend on the returns at t=1, but instead depends on the returns at t=2. In addition, given that B is a small cap stock, if the return on stock B is more than the return on the large cap stock A at t=2, then the size effect is going to be positive.

2.7 Excess portfolio turnover and transaction costs

Since the EWP gets rebalanced after every time period, it will have a higher portfolio turnover compared to that of the VWP. In this section we analyze the excess turnover, denoted by TO_{EX}, defined as the percentage of stocks rebalanced (or traded) to maintain equal weights of stocks in each time period. Since EWP is rebalanced before t=2,

$$TO_{EX} = \frac{((N_{A1ER} + N_{B1ER}) - (N_{A1E} + N_{B1E}))}{(N_{A1E} + N_{B1E})} = \frac{(P_{\mathrm{A1}}P_{B0} - P_{A0}P_{B1})\,(P_{\mathrm{A1}} - P_{\mathrm{B1}})}{2P_{\mathrm{A1}}P_{\mathrm{B1}}\,(P_{\mathrm{A0}} + P_{\mathrm{B0}})} \tag{21}$$

Next, to compute the impact of excess portfolio turnover on portfolio return, we use Aggregate Trading Costs (ATC) based on the methodology and findings of Edelen et al. (2013). ATC captures invisible costs (commission, bid-ask spread, price impact, and the volume of trades) and should be added to the visible cost (expense ratio). ATC is computed by first calculating the per unit cost of a trade and then multiplying the per unit cost of each trade (portfolio change) by the dollar value of the trade and summing across all trades for the time period. As a result of portfolio turnover, the net impact on portfolio return is going to be a negative $TO_{EX} * ATC$.

3 Data

Historical stock-level data are not required in Sections 4.1, and 4.2 because prices are simulated. When individual portfolios are not required as in Section 4.3, and 4.4, VW and EW index returns (including dividends) are used from the CRSP (ASI) dataset for years from 1926 to 2014. To create VWP and EWPs in Section 4.5, stock-level data are obtained from CRSP monthly stock dataset from January 1926 to December 2014. A total of 4,150,448 monthly stock returns were obtained for the analysis.[3]

When investable indexes are required for comparison purposes as in Section 4.7, the Russell 1000 index (which contains 1000 large cap firms with 90% of the total market capitalization of Russell 3000 index) is used as a proxy for large cap returns, and the Russell 2000 index (which contains 2000 small cap firms with 10% of the total market capitalization of Russell 3000 index) is

[3] Since the same ticker symbol is used for multiple companies, and same company can have multiple classes of shares during the window of analysis, PERMCO is used as a unique key for analysis.

used as a proxy for small cap returns. Russell indices monthly data are available between October 1992 and December 2014. Summary statistics of data is shown in Table 1.

Insert Table 1 here.

To ensure that there is enough liquidity in the underlying stocks and to compare performance with the S&P500, the maximum number of stocks in the EWP is limited to the largest 500 stocks by market value. If any company's stock is delisted during the year, that stock position is liquidated using the last available price at the beginning of the year, and those proceeds are allocated either according to the weights (in case of VWP) or equally (in case of EWP). For the VWP, the portfolio is rebalanced at the beginning of every year to account for any new listings and delistings. For the EWP, the portfolio is rebalanced at the beginning of every year to account for any new listings, delistings, and change in portfolio weights during the year.

4 Results

The VW and EW equity portfolio model developed in Section 2 is validated based on the following five tests. For analysis and discussion purposes, the VW return R_{2V} is computed with equation (2), EW return R_{2E} with equation (4), and excess return $R_{2E} - R_{2V}$ with equation (6). Excess risk σ_E-σ_V is computed as described in Section 2.4, and excess Sharpe ratio S_{E-V} with equation (16). The five tests are

Test 1: Random prices, or random returns for small cap and large cap stocks.

Test 2: Normally distributed returns with higher mean and standard deviation for small cap stocks.

Test 3: Bootstrapping simulation using historical data from 1926 to 2014.

Test 4: Curve fitting using historical data from 1926 to 2014.

Test 5: Constructing EWP and VWPs using historical data from 1926 to 2014.

4.1 Random prices

In this test, it is assumed that the initial market cap at t=0 for A is larger than that of B. All the prices $P_{A0}, P_{A1}, P_{A2}, P_{B0}, P_{B1}, P_{B2}$ are randomly chosen between \$0 and \$100 with equal probabilities. Although this type of price pattern is unrealistic in real-world, the purpose of this test is to demonstrate that the excess return $R_{2E} - R_{2V}$ can be positive when stock prices are randomly-generated.

As shown in Figure 1, after 10,000 such simulations, the EWP produces positive returns 59% of the time, whereas VWP produces positive returns 49% of the time. The excess return $R_{2E} - R_{2V}$ is positive 66% of the time, with a mean of 15.5%. The Sharpe ratio is also higher for the EWP. VWP has a mean return of -0.55%, standard deviation of 36.94%, and Sharpe ratio of -0.096. EWP has a mean return of 14.92%, standard deviation of 48.90%, and Sharpe ratio of 0.24. Although the results reported here assume a risk-free rate of 3%, the results are unchanged for any risk-free rate between 0% and 5%. In repeated simulations it is found that the EWP has a higher mean return, higher standard deviation, and higher Sharpe ratio. This case demonstrates that even with totally random prices, a EWP produces superior returns compared to a VWP.

Insert Figure 1 here.

4.2 Normally distributed returns

In this test, returns $R_{A1} = \frac{P_{A1}-P_{A0}}{P_{A0}}$, and $R_{A2} = \frac{P_{A2}-P_{A1}}{P_{A1}}$ are normally distributed for a large cap stock, with a mean and standard deviation of (μ_L, σ_L). Similarly, returns $R_{B1} = \frac{P_{B1}-P_{B0}}{P_{B0}}, R_{B2} = \frac{P_{B2}-P_{B1}}{P_{B1}}$ are normally distributed for a small cap stock, with a mean and standard deviation of (μ_S, σ_S). Given that small cap returns are larger and more volatile than the same for large cap, it is quite natural to have $\mu_L < \mu_S$, and $\sigma_L < \sigma_S$. For the purpose of our illustration, the results of 10,000 such excess return simulations for $\mu_L = 5\%, \mu_S = 8\%, \sigma_L = 10\%, \sigma_S = 15\%$ are shown in Figures 2, 3, and 4. However, the results are tested for robustness for any $\mu_L < \mu_S$, and $\sigma_L < \sigma_S$.

The EWP produces positive returns 76% of the time, whereas VWP produces positive returns 75% of the time. The mean excess return $R_{2E} - R_{2V}$ is 0.79%, and it is positive in 56.4% of the simulations. The mean, and median excess returns are also positive. This pattern is consistent in repeated simulations for any $\mu_L < \mu_S$, and $\sigma_L < \sigma_S$.

The Sharpe ratio is also higher for the EWP. VWP has a mean return of 5.8% and standard deviation of 8.46%, whereas EWP has a mean return of 6.6% and standard deviation of 9.0%. Assuming a risk-free rate of 3%, the Sharpe ratio is 0.33 for VWP and 0.40 for EWP.

Insert Figures 2, 3, 4 here.

In summary, both the random prices in Section 4.1, and normally distributed returns in Section 4.2 illustrate that EWP outperforms VWP in both mean return and Sharpe ratio. In the next section historical data are used rather than simulated random prices and simulated normal returns.

4.3 Bootstrapping historical data

In this test, VW and EW annual returns (including dividends) are obtained from CRSP (ASI) dataset for the years from 1926 to 2014. A summary of these two returns are shown in Figures 5, 6, and Table 1.(2).

Insert Figures 5 and 6 here.

To compute a 2-period return, any two years are randomly selected using replacement from 1926 to 2014. Then for each selected year, the corresponding VW and EW returns are used from the dataset. By doing so, the R_{A1}, R_{A2}, R_{B1}, R_{B2} used in Section 4.2 are no longer selected from a normal distribution. Instead they are picked from the past historical data. This process was repeated 10,000 times.

The results are summarized in Figures 7, 8, and 9. After 10,000 such simulations, EWP produces positive returns 75.2% of the time, whereas VWP produces positive returns 73.9% of the time. The excess return $R_{2E} - R_{2V}$ is positive 54.1% of the time, has mean of 1.01%, and similar to the 1.8% annual excess return found by the S&P Indices global research team over a 20-year period between 1998 and 2008.[4] In addition, the Sharpe ratio of the EWP is also slightly higher than the VWP (0.46 vs 0.45), again assuming a risk-free rate of 3%. Given the similarity of EWP and VWP returns, we conducted a pair-wise t-test on EWP and VWP to check the statistical significance of results. The results are shown in Table 2. A p-value of 0.00 suggests that EW returns are statistically different to VW returns, and the 95% confidence interval shows that excess returns are positive. As in the previous two test, in this test the EWP produces statistically significant and better excess return and Sharpe ratios.

Insert Figures 7, 8, 9, and Table 2 here.

[4] `https://us.spindices.com/documents/research/EqualWeightIndexing_7YearsLater.pdf`

4.4 Curve fitting historical data

In this test, VW and EW annual returns (including dividends) are used from CRSP (ASI) dataset for years from 1926 to 2014. The actual return distribution is fitted to the best possible matching distribution using the Anderson-Darling (A-D) method. Using this approach, the best match for large cap stocks with an A-D of 0.1741 is a Weibull distribution with a location of -101.10%, scale of 120.97%, and shape of 6.672. Similarly, the best match for small cap stocks with an A-D of 0.2377 is a logistic distribution with a mean of 15.56%, and scale of 15.29.

Next, using these matching parameters, returns are computed for EWP and VWPs using 10,000 simulations. As shown in Figure 10, the EWP produces positive returns 79.4% of the time, whereas VWP produces positive returns 78.1% of the time. The annual excess return $R_{2E} - R_{2V}$ is positive 54.8% of the time with a mean of 1.03%. In addition, the Sharpe ratio of the EWP is also higher than the VWP (0.63 vs 0.58).

Insert Figure 10 here.

4.5 Constructed portfolios

In this section, EWP and VWPs are constructed consisting of all the publicly traded stocks using monthly historical data from 1926 to 2014. To ensure that there is enough liquidity in the underlying stocks and to compare performance with the S&P500, the maximum number of stocks in the EWP is limited to the largest 500 stocks by market value. If any company's stock is delisted during the year, that stock position is liquidated using the last available price at the beginning of the year, and those proceeds are allocated either according to the weights (in case of VWP) or equally (in case of EWP). For the VWP, the portfolio is rebalanced at the beginning of every year to account for any new listings and delistings. For the EWP, the portfolio is rebalanced at the beginning of every year to account for any new listings, delistings, and change in portfolio weights during the year.

After rebalancing every year, returns are computed for EWP and VWPs. The results of a pair-wise t-test on EW and VW constructed portfolio returns are shown in Tables 3 and 4. Between the years 1926 and 2014, EWP produced positive returns 60.2% of the time, whereas VWP produced positive returns 56.8% of the time. The annual excess return $R_{2E} - R_{2V}$ is positive 69.3% of the time, with a mean of 4.1%. In addition, the Sharpe ratio of the EWP is also higher than the VWP (0.11 vs -0.07).

Insert Tables 3 and 4 here.

In conclusion, in the absence of trading costs, in all the five tests (i.e.,: theoretical random prices, normally distributed returns, matching historical index prices using bootstrapping simulation, simulated returns using historical rates using curve fitting, and actual portfolio construction), EWP returns are higher and positive more number of times when compared to the VWP returns. In addition, Sharpe ratios are higher for EWPs. The results from five tests are summarized in Table 4.

4.6 Excess risk

In this section, we switch our attention from return to risk in order to determine if the excess return is obtained by taking excess risk, and, if so, what happens to the risk-adjusted returns. In the context of EWP and VWPs, excess risk is defined for a given time period as the EWP standard deviation minus the VWP standard deviation. Excess risk is computed as explained in Section 2.4 using equation (9).

The distribution of d, e, ρ, σ_L in this section is based on the historcial VW and EW annual return data (including dividends) obtained from the CRSP (ASI) dataset for the years from 1926 to 2014. The following parameters characterize the underlying data: d is between 5%, and 15% with equal probability, e is between 5% and 45% with equal probability, ρ is between -0.9 and 0.9 with a likely value of 0.6 using a triangular distribution, and σ_L is between 10% and 25% with equal probability. All these parameters, small cap return R_S, and large cap return R_L are selected using bootstrap by replacement method as described in Section 4.3.

After 10,000 such simulations, the excess risk is found to be positive 99% of the time. It simply means that EWP produces excess return over VWP, but as one would expect, at the cost of higher risk. However, as shown in Table 4, and explained in Section 4.7, the Sharpe ratio is always higher for the EWPs. As shown in Figure 11, the excess risk varies from -2.18% to 4.37%, with a mean of 1.15%, a median of 1.01%, and a standard error of 0.01%. The excess risk is negative only when the large cap weight approaches 100% of the portfolio.

Insert Figure 11 here.

4.7 Excess Sharpe Ratio

In the context of EWP and VWPs, the excess Sharpe ratio is defined for a given time period as the EWP Sharpe ratio minus the VWP Sharpe ratio. As explained in Section 2.5, excess Sharpe ratio is computed using equation (16).

The distribution of d, e, ρ, σ_L in this section is the same as the one described in Section 4.6. Although the results reported here assume a risk-free rate of 3%, the results do not change significantly for any risk-free rate between 0% and 5%.

As shown in Figure 12, after 10,000 simulations the excess Sharpe ratio of EWP over VWP is found to be positive 63% of the time, and the average varies from 0.009 for small equal-weighting effect to 0.06 for large equal-weighting effect. The average excess Sharpe ratio for the entire dataset is 0.03. This is close to the observed excess Sharpe ratio of 0.04 between two of the largest exchange-traded funds, Guggenheim S&P 500 Equal Weight ETF (RSP)[5], and SPDR S&P 500 ETF (SPY).[6] As of July 2015, the Sharpe ratio for RSP is 1.98 and 1.94 for SPY.

For robustness, we computed the 95% confidence interval of the mean excess Sharpe ratio. It is between 0.0272 and 0.0331 with a t-statistic of 20. This clearly validates that the risk-adjusted return of the EW portfolios is higher than that of the VW portfolios. Though in aggregate the excess Sharpe ratio is positive, as one would expect, during some periods it can be negative. As an example during the credit crisis (years 2007 and 2008) as well as during the dot-com bubble (years 1999, 2000, 2001) excess Sharpe ratio is positive. However post credit crisis (years 2009 to 2014) excess Sharpe ratio is negative. Parameter values for these sub-periods can be found in Table 1.

Insert Figure 12 here.

4.8 Decomposition of excess return into rebalance and size effect

After showing that excess returns are positive, the next question to answer is about the source of these excess returns. As described in Section 2.6, the excess return is decomposed into return due to rebalancing, and return due to size (small-cap). Although Perold and Sharpe (1995) did

[5] `http://finance.yahoo.com/q/rk?s=RSP`
[6] `http://finance.yahoo.com/q/rk?s=SPY`

not use the word equal-weighting, they recognized that periodic rebalancing of a portfolio to its target allocation as one of the dynamic strategies for asset allocation. What they called Constant Proportion Portfolio Insurance (CPPI) can be thought of as a variation of EWP. CPPI was found to outperform other dynamic strategies in a bull market and simpler to implement than an option-based portfolio insurance.

In this section, excess return $R_{2E} - R_{2V}$ is decomposed into return due to rebalancing $R_{2E} - R_{2E\_NR}$ using equation (19), and return due to size effect $R_{2E\_NR} - R_{2V}$ using equation (20). We present here the results from the Section 4.1 (random prices), as they are most generic and not dependent on any index or market condition, to illustrate the point that significant portion of excess returns are due to rebalancing of the portfolio.

In each of the 10,000 simulation runs, the excess return is decomposed into return due to rebalancing, and return due to size effect. The results are summarized in Figure 13. In 19 out of 20 categories, rebalancing is the main contributor to the excess return. On average, 85% of the excess returns are due to rebalancing effect. In addition, it can be seen in Figure 14 that almost all the average excess return $R_{2E} - R_{2V}$ of 15.64% is due to the return from the equal-weighting (R_{2E} of 15.97%), with very little attributable to the return from value-weighting (R_{2V} of 0.34%). Almost all the excess return is due to the rebalancing effect of 15.67%.

Insert Figures 13, and 14 here.

4.9 Excess portfolio turnover and transaction costs

As explained in Section 2.7, excess turnover (TO_{EX}) is defined as the percentage of stocks traded to rebalance and maintain equal weights of stocks in each time period.

As the portfolio becomes more equal-weighted, portfolio turnover increases and excess return also increases. As shown in Figure 15, for every 1% increase in portfolio return the portfolio turnover increases by 0.4669%. The mean of TO_{EX} is 13.61% with a standard deviation of 25.35%.

Next, to compute the impact of excess portfolio turnover on portfolio return, we use ATC based on the methodology and findings of Edelen et al. (2013). They calculated ATC as 1.69% for large funds (with an average of $2.88 billion assets) and 1.19% for small funds (with an average of $164 million assets), based on the 3,799 open-end domestic equity mutual funds data using quarterly portfolio holdings data from Morningstar from 1995 to 2006. Fund size plays a role in trading costs because estimated per unit trading costs are more than 30 bps higher for large funds than for small funds. As a result of portfolio turnover, the net impact on annual portfolio return is going to be a negative $TO_{EX} * ATC$, or -0.23% for large funds and -0.16% for small funds. As shown in Table 4, in all five tests the mean of excess returns is higher than 0.23%. Because the benefit of equal-weighting is higher than the cost, equal-weighting makes economic sense even after accounting for higher portfolio turnover costs.

Insert Figure 15here.

5 Conclusion

Recent studies by DeMiguel et al. (2009) and Plyakha et al. (2015) suggest that equal-weighted portfolios (also known as 1/N, sometimes abbreviated as EWP) appear to outperform 14 different portfolio weighting strategies. Several portfolio weighting strategies, including alternative beta strategies[7] have emerged in the marketplace, as they all seem to outperform VW indices according to Arnott et al. (2005), Chow et al. (2011), and Amenc et al. (2011a). Given the positive excess returns of EWPs over VWPs in the equity asset class, two natural questions arise. First, Can this excess return be realized under various market scenarios and time horizons? Second, what is the source of these positive excess returns? In this paper we provide a theoretical framework for the EWP model, and demonstrate using simulation and historical data that an EW strategy indeed outperforms VW strategies. We then show that rebalancing is a key driver behind the positive excess retun of the EWPs.

To undertake our analysis, we first develop a EWP and VWP model that allows for portfolio rebalancing. This model demonstrates that after rebalancing a portfolio, if the smaller-cap stocks outperform the larger-cap stocks, then the EWP will produce higher returns than a VWP. We use five tests to confirm that the EWP outperforms the VWP in terms of return and Sharpe ratio.

In order to understand the source of the positive excess return of EWP over VWP, we decompose the excess return into return due to rebalancing effect, and return due to size effect. The rebalancing

[7] http://www.edhec-risk.com/edhec_publications/all_publications/RISKReview.2015-03-26.2929/attachments/EDHEC_Publication_Alternative_Equity_Beta_Investing_Survey.pdf

effect does not depend on the market cap of stocks in the portfolio, and dependents solely on price movements. The size effect depends on both price fluctuations and the market cap of stocks. We show that in 19 out of 20 categories, rebalancing is the main contributor to the excess return. On average, 85% of the excess returns are due to rebalancing effect, and the remaining 15% are due to size effect.

Finally, we show that because of equal-weighting the excess returns are higher than the higher costs incurred due to higher portfolio turnover. Therefore, even after accounting for higher portfolio turnover costs, equal-weighting makes economic sense.

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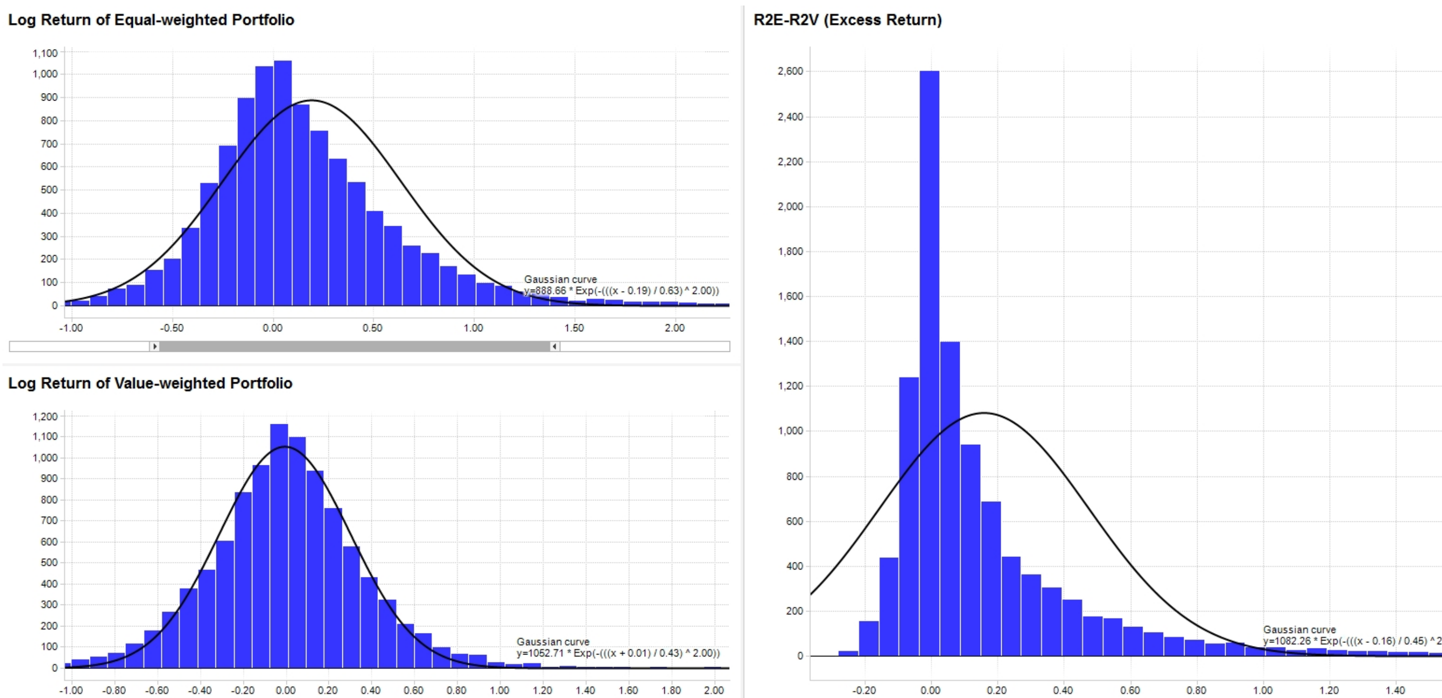

Figure 1. Simulated excess return $R_{2E} - R_{2V}$ using random prices $P_{A0}, P_{A1}, P_{A2}, P_{B0}, P_{B1}, P_{B2}$. After 10,000 simulations, EWP produces positive returns 59.2% of the time, whereas VWP produces positive returns 49.2% of the time. This pattern is consistent in repeated simulations. The excess return $R_{2E} - R_{2V}$ is positive 65.5% of the time. The Gaussian fitted curve shows a mean return of 19% for EWP, and -1% for VWP. More detailed statistics can be seen in Table 4

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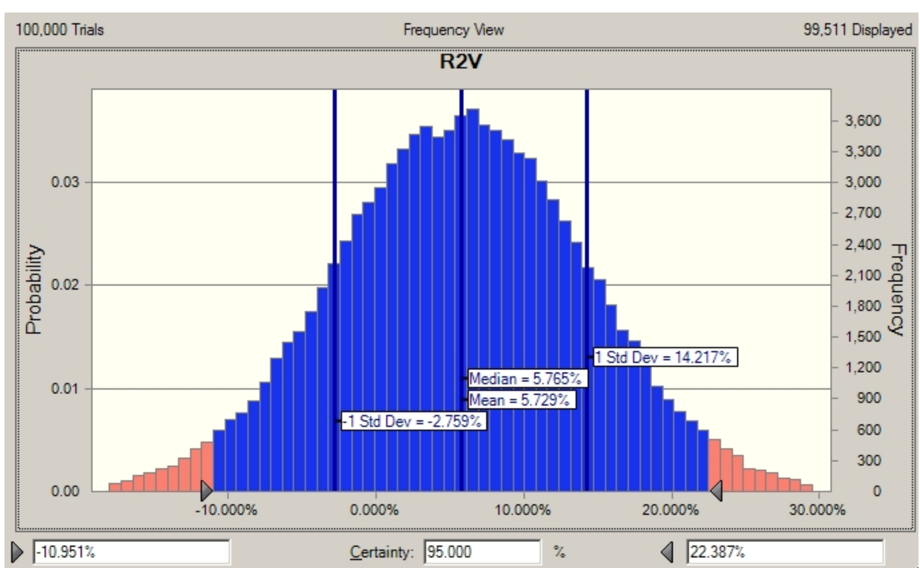


Figure 2. Simulated VW return R_{2V} using normal returns $\mu_L = 5\%, \mu_S = 8\%, \sigma_L = 10\%, \sigma_S = 15\%$.

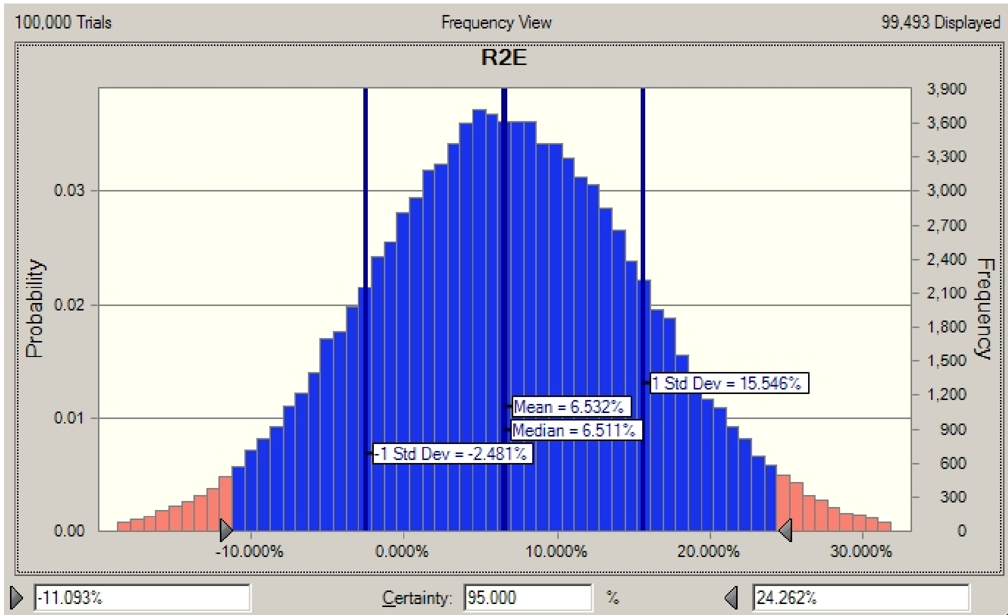


Figure 3. Simulated EW return R_{2E} using normal returns $\mu_L = 5\%, \mu_S = 8\%, \sigma_L = 10\%, \sigma_S = 15\%$.

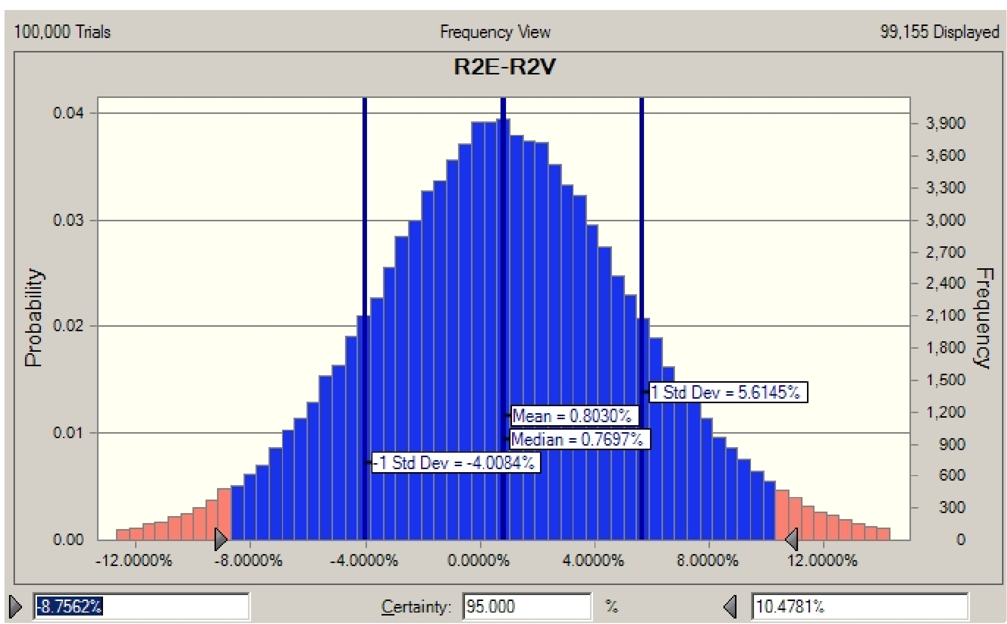


Figure 4. Simulated excess return $R_{2E} - R_{2V}$ using normal returns $\mu_L = 5\%, \mu_S = 8\%, \sigma_L = 10\%, \sigma_S = 15\%$. After 10,000 such simulations, the excess return is positive 56.4% of the time. The mean, and median excess returns are also positive. More detailed statistics can be seen in Table 4.

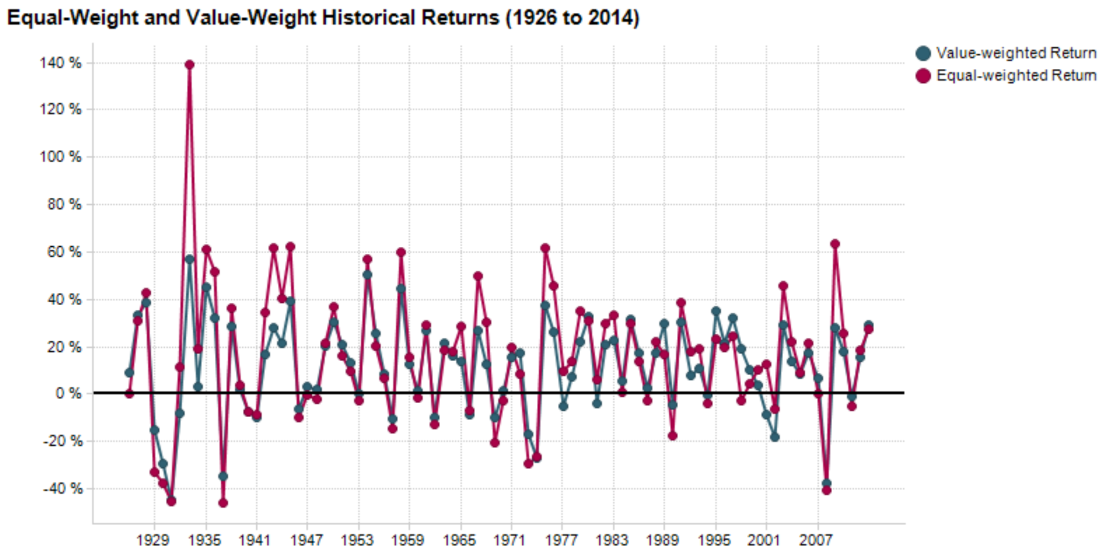


Figure 5. Historical returns of Value-weighted (top 500 VW) and Equal-weighted (bottom 2000 by EW) index.

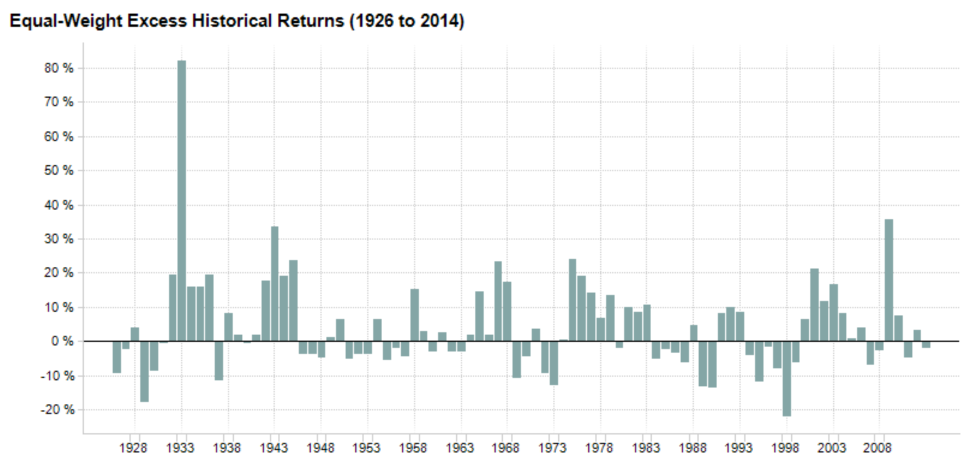


Figure 6. Excess returns (equal-weight minus value-weight) using historical data (1926-2014).

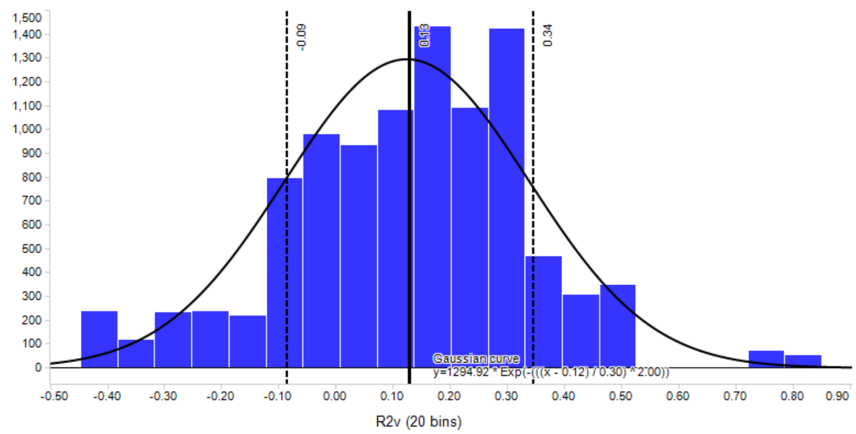


Figure 7. Value-weighted returns R_{2V} using historical data (1926-2014) and Bootstrapping simulation. Annualized VW mean return for the period is 13%, with a standard deviation of 21%.

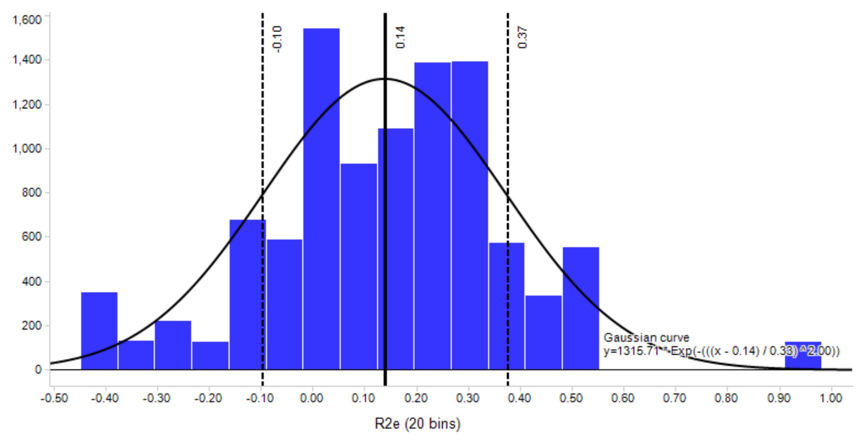


Figure 8. Equal-weighted returns R_{2E} using historical data (1926-2014) and Bootstrapping simulation. Annualized EW mean return for the period is 14%, with a standard deviation of 23%.

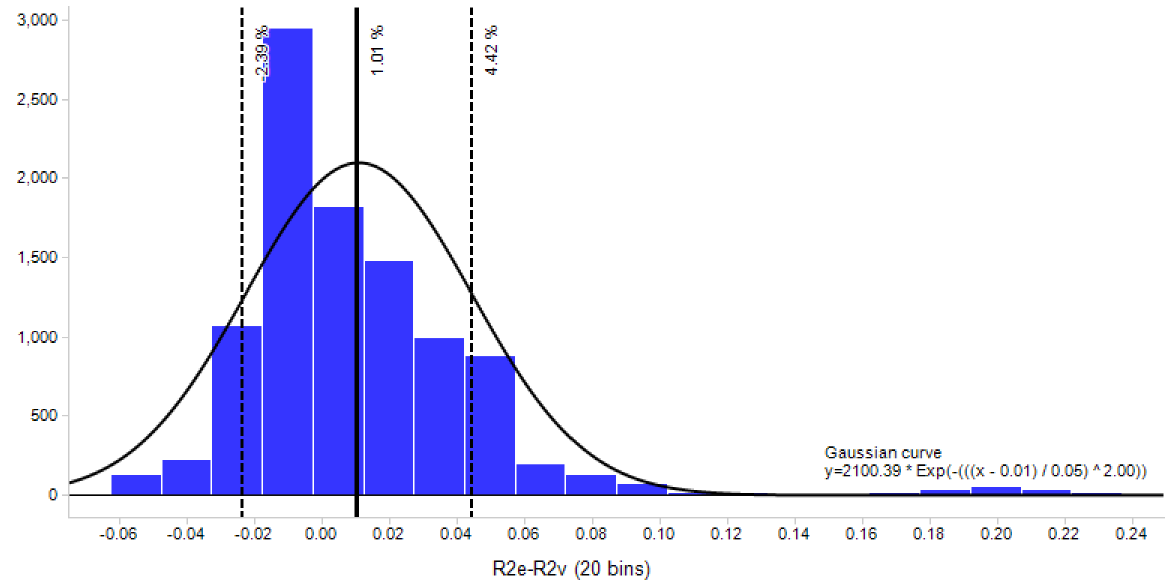


Figure 9. Excess returns $R_{2E} - R_{2V}$ using historical data (1926-2014) and Bootstrapping simulation. After 10,000 simulations, annualized EW mean return for the period is 1.01%, with a standard deviation of 3.41%.. In addition, the Sharpe ratio of the EWP is also higher than the VWP (0.461 vs 0.457), and a positive skewness of 2.23.

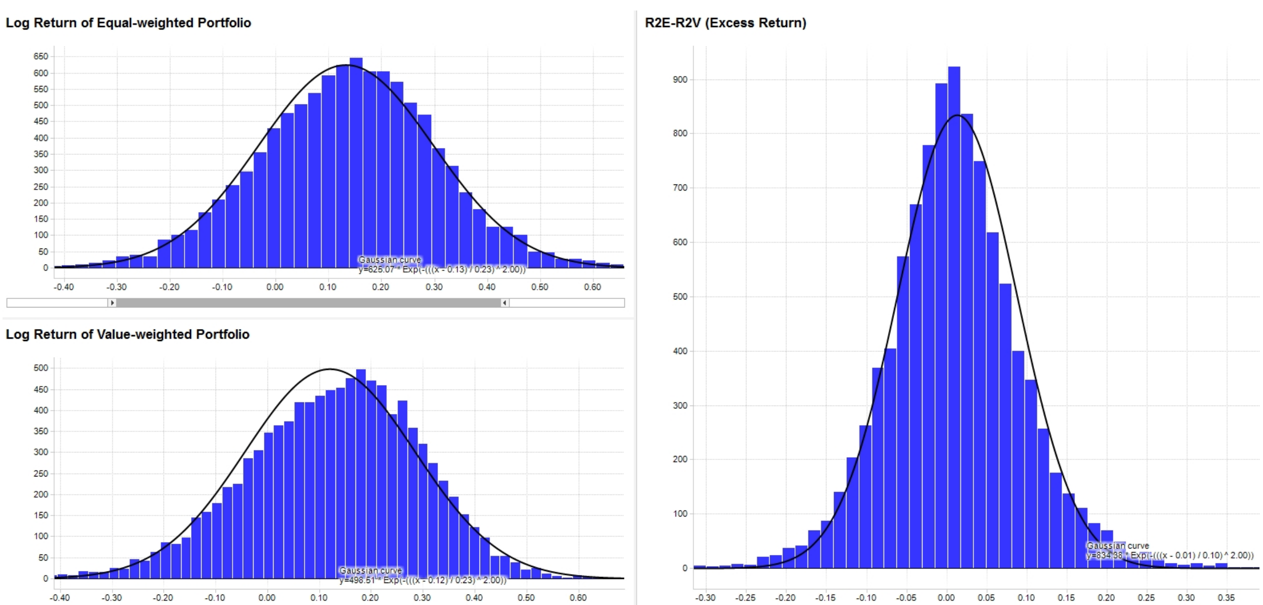


Figure 10. Excess returns $R_{2E} - R_{2V}$ using historical data (1926-2014) and curve fitting. After 10,000 runs, the EWP produced 1.03% more annual return than the VWP. The excess return has a positive skewness of 0.14. In addition, the Sharpe ratio of the EWP is also higher than the VWP (0.63 vs 0.58).

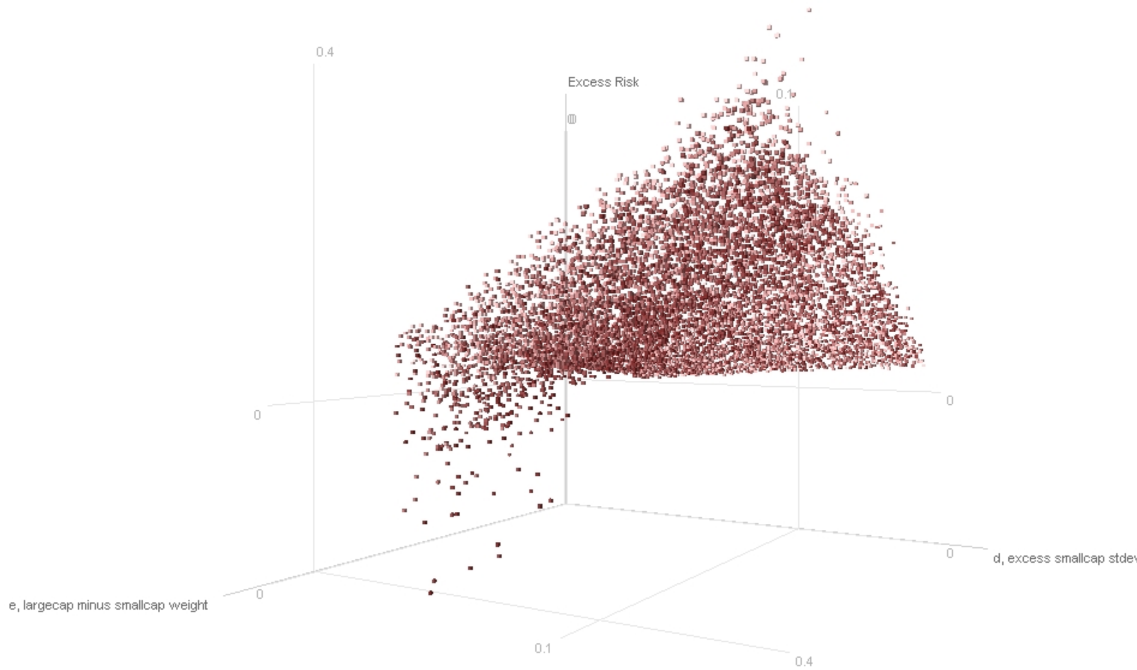


Figure 11. Excess risk of EWP over VWP as d and e change. The excess risk is positive 99% of the time and it varies from -2.18% to 4.37%, and has a mean of 1.15%, a median of 1.01%, and a standard error of 0.01%. The excess risk is negative only when the large cap weight approaches 100% of the portfolio (shown in lower left part of the chart).

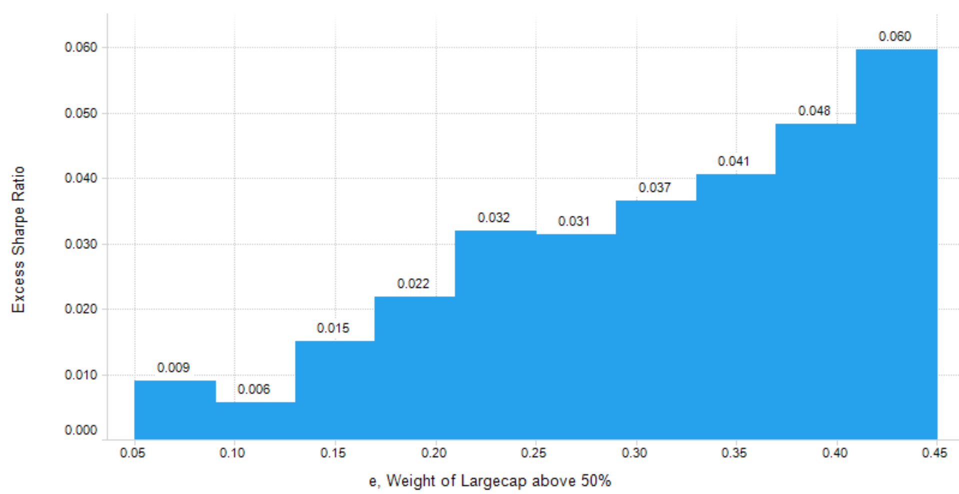


Figure 12. Excess Sharpe ratio of EWP over VWP. The excess Sharpe ratio is positive 63% of the time and the average varies from 0.009 for small equal-weighting effect to 0.06 for large equal-weighting effect. When e increases (move rightward on the x-axis), it means large cap weight is increasing in the portfolio. In other words, the portfolio is deviating more from an equal-weight portfolio. That means equal-weighting effect is going to be more noticeable. The average excess Sharpe ratio for all the 10,000 simulations is 0.03. The 95% confidence interval of the mean excess Sharpe ratio is between 0.0272 and 0.0331 with a t-stat of 20.

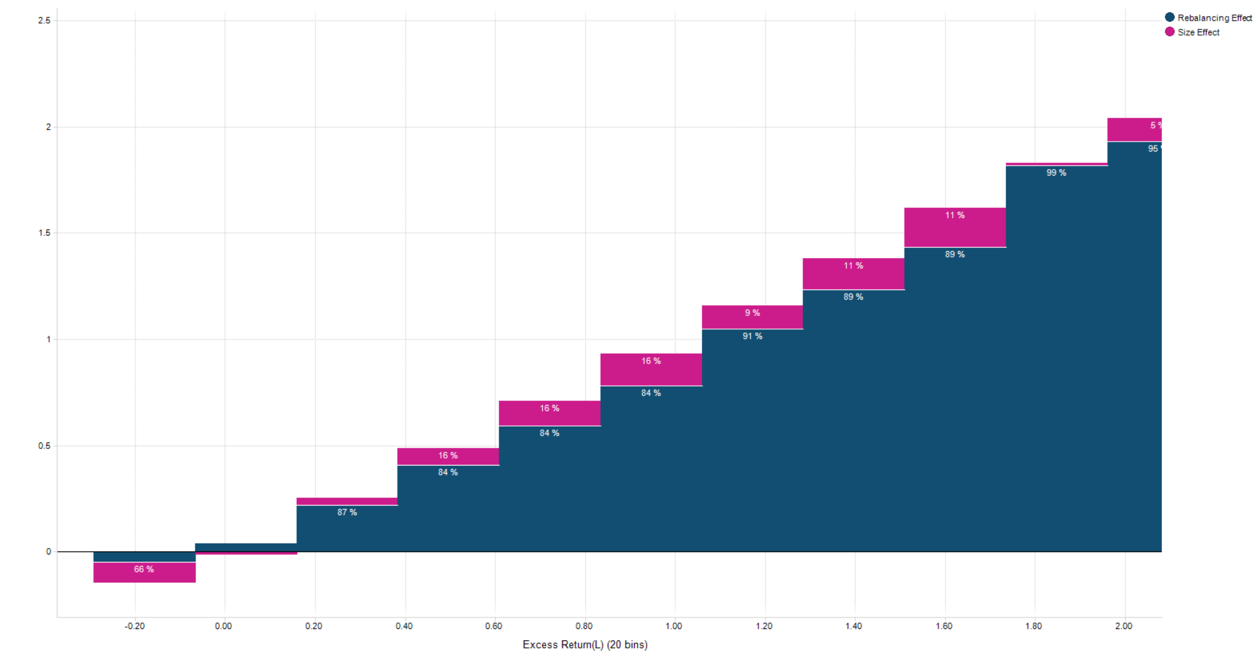
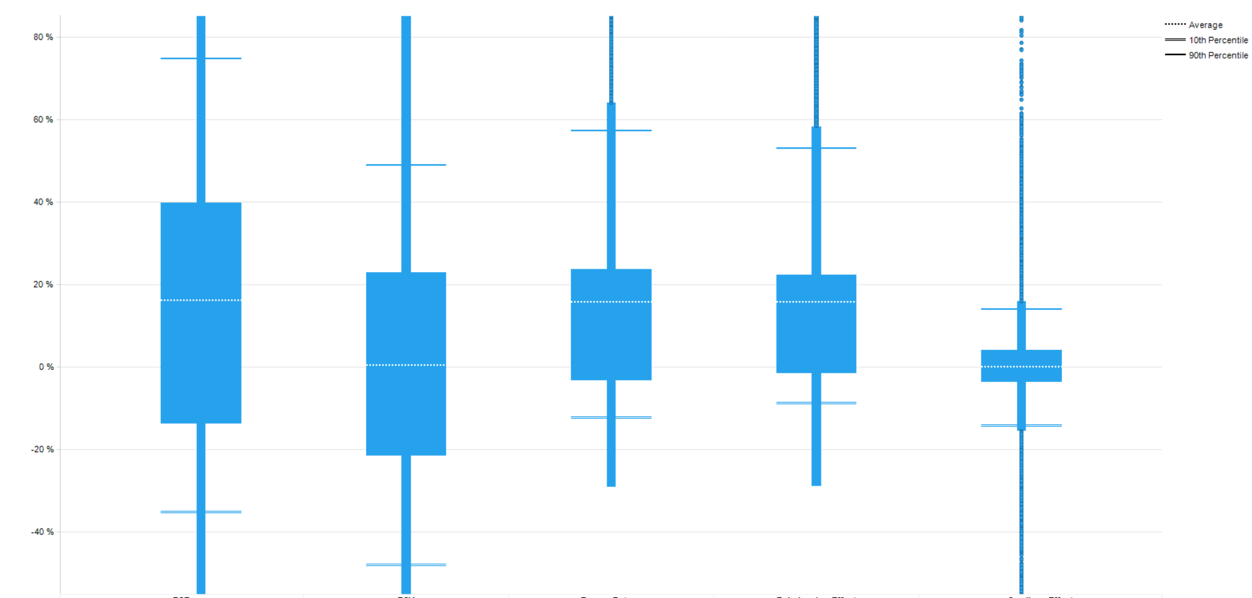


Figure 13. Decomposition of excess returns $R_{2E} - R_{2V}$ (x-axis) into return due to rebalancing effect $R_{2E} - R_{2E\_NR}$ (shown in lower bars) using equation (19), and return due to size effect $R_{2E\_NR} - R_{2V}$ (shown in upper bars) using equation (20). As shown in this figure, 85% of the excess returns are due to rebalancing effect.



	R2E	R2V	Excess Return Current filtering	Rebalancing Effect	Smallcap Effect
Count	10,000	10,000	10,000	10,000	10,000
Avg	15.97 %	0.34 %	15.64 %	15.67 %	-0.03 %
StdDev	48.97 %	41.19 %	34.84 %	33.04 %	16.10 %

Figure 14. In aggregate, almost all the excess return $R_{2E} - R_{2V}$ of 15.64% is due to the EWP return R_{2E} of 15.97% over VWP return R_{2V} of 0.34%. In addition, almost all the excess return is due to the rebalancing effect of 15.67%.

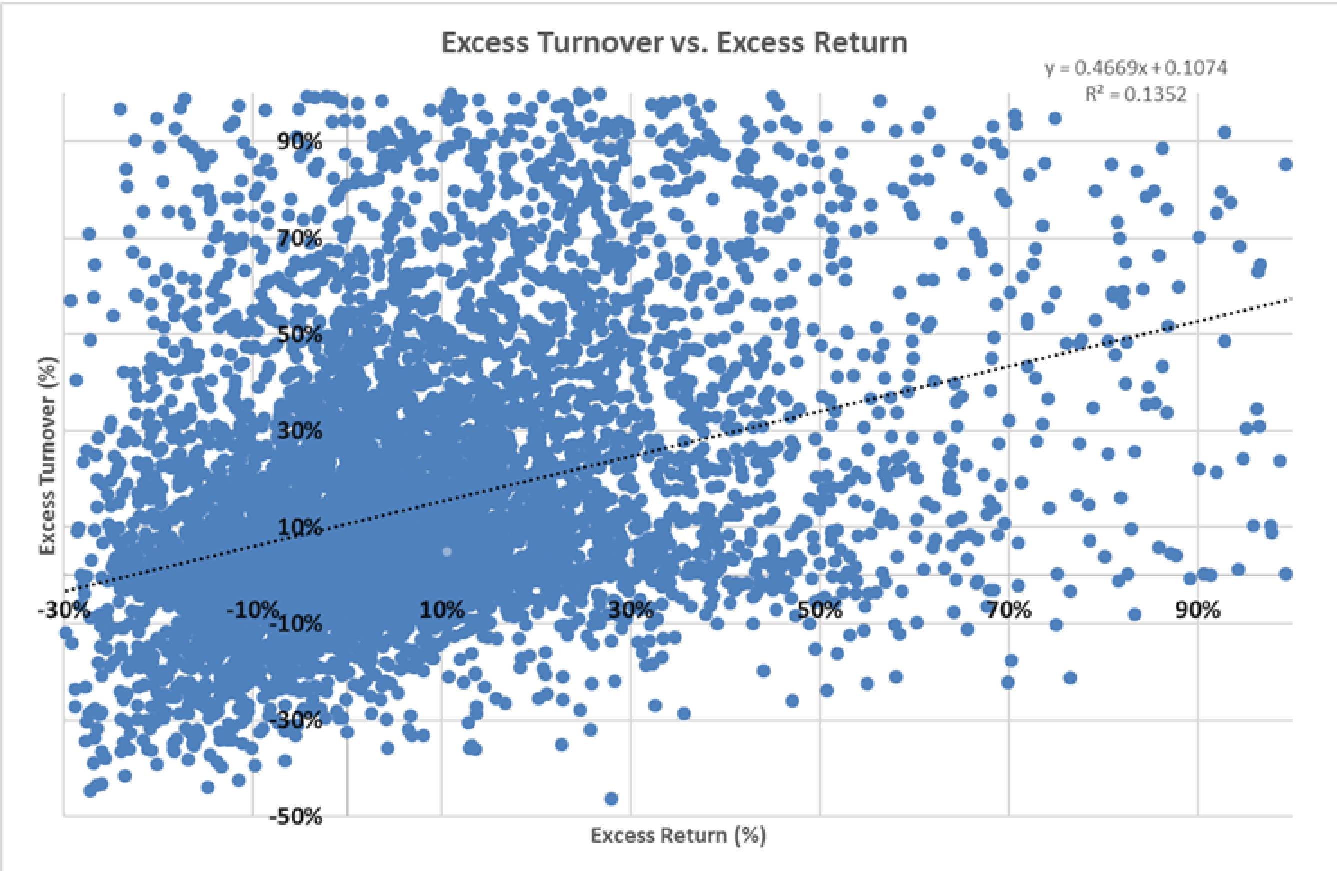


Figure 15. Portfolio excess return results versus excess portfolio turnover using random prices test as explained in section 4.1. As portfolio turns more EW from VW, portfolio turnover increases and excess return also goes up. For every 1% increase in portfolio return, portfolio turnover increases by 0.4669%. The mean portfolio turnover is 13.61% with a standard deviation of 25.35%.

(1) Small cap and large cap returns	N	Mean(μ)	Median	Stdev (σ)	L95	U95	
Small cap monthly return (Russell 2000). 12/1992 to 12/2014	264	0.80%	1.60%	5.52%	0.13%	1.46%	
Large cap monthly return (Russell 1000). 12/1992 to 12/2014	264	0.70%	1.17%	4.28%	0.18%	1.21%	
(2) Equal-weight and Value-weight index returns							
Equal-weighted CRSP monthly returns (01/1926 to 12/2014)	1,068	1.15%	1.41%	6.85%	0.74%	1.56%	
Value-weighted CRSP monthly returns (01/1926 to 12/2014)	1,068	0.94%	1.30%	5.49%	0.61%	1.27%	
Stock prices used in EW-VW portfolios (monthly data, 12/1926 to 12/2014))	4,150,448						
Relationship between datasets	P-value	F-stat	R-Square	Corr. (ρ)	R_L	R_S	
Relationship between Russell 2000 and Russell 1000	0.00	603.73	0.70	0.84			
Relationship between CRSP VW and CRSP EW	0.00	11,424	0.91	0.96			
(3) Scenarios for large cap and small cap (based on Russell 2000 and 1000)	N	σ_L	σ_S	d	Corr. (ρ)	R_L	R_S
All months from 10/1992 to 12/2014	264	4.27%	5.52%	1.24%	0.84	0.70%	0.80%
Years 2007 and 2008	24	5.24%	6.38%	1.14%	0.94	-1.74%	-1.67%
All positive months	161	2.70%	3.03%	3.33%	0.55	2.95%	4.21%
All negative months	103	3.81%	3.86%	4.91%	0.78	-2.82%	-4.54%
Years 2000 and 2001	24	5.36%	7.41%	2.05%	0.63	-0.85%	0.13%
Years 2009 to 2014	72	3.83%	5.19%	1.35%	0.93	1.12%	1.23%

Table 1. Summary statistics include the following.

(1) Russell 2000 and 1000 monthly stock returns from 12/1992 to 12/2014, as proxies for small cap and large cap.

(2) CRSP equal-weight and value-weight index returns from 01/1926 to 12/2014.

(3) Different scenarios tested on Russell 2000 and 1000 monthly stock returns from 12/1992 to 12/2014, as proxies for small cap and large cap.

Paired T-test for EWP and VWP				
	N	**Mean**	**StDev**	**SE Mean**
R_{2E}	10,000	0.13883	0.23613	0.00236
R_{2V}	10,000	0.12869	0.21582	0.00216
$R_{2E} - R_{2V}$	10,000	0.01014	0.03405	0.00034
95% confidence interval for $R_{2E} - R_{2V}$			(0.00947, 0.01081)	
T-Test of mean of difference = 0			(vs. #0)	
P-value: 0.000		T-value: 29.78		

Table 2. Results of matching historical data using Bootstrapping simulation. Pair-wise summary statistics of EW return R_{2E}, VW return R_{2V}, and excess return of EWP $R_{2E} - R_{2V}$ show a positive excess return. A p-value of 0.000 suggests that EW returns are statistically different to VW returns using the data from years 1926 to 2014.

Paired T-test for EWP and VWP				
	N	**Mean**	**StDev**	**SE Mean**
R_{2E}	88	0.0572	0.2502	0.0267
R_{2V}	88	0.0165	0.1931	0.0206
$R_{2E} - R_{2V}$	88	0.0407	0.0955	0.0102
95% confidence interval for R2E-R2V			(0.0204, 0.0609)	
T-Test of mean of difference = 0			(vs. #0)	
P-value: 0.000		T-value: 3.99		

Table 3. Constructed portfolio statistics of EW return R_{2E}, VW return R_{2V}, and excess return of EWP $R_{2E} - R_{2V}$ using the data from years 1926 to 2014. During this period EWPs produced statistically significant excess annual return of 4.07%.

	1) Random Prices			2) Normal Returns			3) MonteCarlo			4) Curve Fitting			5) Constructed Portfolios		
	R2E	*R2E-R2V (log)*	*R2V*	*R2E*	*R2E-R2V*	*R2V*	*R2E*	*R2E-R2v*	*R2V*	*R2E*	*R2E-R2V*	*R2V*	*R2E*	*R2E-R2V*	*R2V*
Mean	14.92%	**15.48%**	-0.55%	6.59%	**0.79%**	5.81%	13.88%	**1.01%**	12.87%	13.70%	**1.03%**	12.67%	5.7%	**4.1%**	1.6%
Median	8.49%	4.95%	-0.28%	6.64%	0.74%	5.74%	15.73%	0.39%	15.70%	14.16%	0.81%	13.67%	6.3%	2.8%	2.5%
Standard Error	0.49%	0.31%	0.37%	0.09%	0.05%	0.08%	0.24%	0.03%	0.22%	0.17%	0.08%	0.17%	2.7%	1.0%	2.1%
Standard Deviation	48.90%	31.25%	36.94%	9.04%	4.89%	8.46%	23.61%	3.41%	21.58%	17.09%	8.39%	16.53%	25.0%	9.6%	19.3%
% of positive returns	59.29%	**66.46%**	49.59%	76.30%	**56.40%**	75.40%	75.20%	**54.10%**	73.90%	79.40%	**54.80%**	78.10%	60.20%	**69.30%**	56.80%
Kurtosis	3.20	14.15	1.83	(0.08)	0.47	0.06	1.21	9.97	0.56	0.51	1.43	0.05	5.35	14.38	1.15
Skewness	0.94	3.01	(0.10)	0.01	0.02	0.03	0.13	2.23	(0.18)	(0.07)	0.13	(0.28)	1.26	2.92	0.19
Range	6.53	4.08	4.37	0.66	0.49	0.61	1.43	0.30	1.30	1.56	0.95	1.19	1.80	0.72	1.15
Minimum	(2.69)	(0.27)	(2.69)	(0.27)	(0.27)	(0.22)	(0.45)	(0.06)	(0.45)	(0.64)	(0.48)	(0.52)	(0.57)	(0.12)	(0.53)
Maximum	3.83	3.81	1.68	0.40	0.22	0.39	0.98	0.24	0.85	0.92	0.48	0.67	1.23	0.61	0.62
Sharpe Ratio (rf=3%)	**0.244**		**(0.096)**	**0.397**		**0.332**	**0.461**		**0.457**	**0.626**		**0.585**	**0.109**		**(0.070)**
# of observations	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	88	88	88

Table 4. Summary of EW R_{2E}, VW R_{2E}, and excess returns $R_{2E} - R_{2V}$ computed using all five methods (i.e.: theoretical random prices, normally distributed returns, matching historical index prices using Bootstrapping simulation, simulated returns using historical rates using curve fitting, and actual portfolio construction). In all five methods, when compared to VWP returns, EWP returns are higher and positive more number of times. In addition, Sharpe ratios are higher for EWPs.