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Tactical Return, Strategic Return, and Diversification Return

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Abstract

We decompose the geometric average return of an actively-managed portfolio into three well-defined components: tactical return, strategic return, and diversification return. Only the tactical return should be credited to the tactical decisions of the portfolio manager. We apply this formalism to portfolios with both periodic and nonperiodic trading.

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The return of a portfolio of assets generally has two components. One component is due to the selection and weighting of the assets themselves. The other component is due to the decisions of the portfolio manager regarding the buying and selling of assets. Superior investment returns require skill and/or luck in one or both of these activities.

Consider an actively-managed portfolio composed of one asset class, say large cap equities. The usual measure of success of such a portfolio is its performance relative to an index composed of similar assets, say the S&P 500 Total Return index. However, this does not reveal how much of the success of the portfolio is due to the composition of the portfolio relative to the index, and how much is due to the portfolio manager's tactical trading decisions.

Similarly, consider an actively-managed portfolio constructed from a variety of different asset classes. The portfolio manager may change the allocation to the various asset classes depending on their view of economic conditions, an activity termed tactical asset allocation. Since the asset-class weights vary, there is no policy portfolio upon which to build a benchmark. In practice, a wide variety of benchmarks are used, sometimes even more than one for a given portfolio.[2] Regardless of the chosen benchmark, the portfolio's success relative to that benchmark does not reveal how much of that success is due to the construction of the portfolio, and how much is due to the tactical trading decisions of the portfolio manager.

[2] Using two of the largest tactical asset allocation mutual funds for example, Pimco All Asset Fund (PAAIX) has a primary benchmark of the Bloomberg Barclays US TIPS 1-10 Year Index and a secondary benchmark of the Consumer Price Index + 500 Basis Points; Columbia Adaptive Risk Allocation Fund (CRAAX) has a blended benchmark consisting of 60% MSCI ACWI Index and 40% Bloomberg Barclays Global Aggregate Bond Index.

Furthermore, consider an actively-managed portfolio in comparison to a portfolio that is rebalanced to fixed weights. A rebalanced portfolio has no tactical component, as the asset weights are kept fixed. The return of a rebalanced portfolio can be divided into a strategic return, which is due entirely to the returns of the assets, and a diversification return, which is due to the rebalancing of a portfolio of volatile assets. This raises the question of how to define a strategic return and a diversification return for an actively-managed portfolio.

In this paper we develop a method to decompose the return of an actively-managed portfolio into three well-defined components: tactical return, strategic return, and diversification return. The tactical return is due entirely to the portfolio manager's tactical decisions. The strategic return is due entirely to the returns of the assets. The diversification return, which was originally derived by Booth and Fama [1992] for a rebalanced portfolio with fixed weights, is generalized to an actively-managed portfolio. Thus we construct an objective method to attribute the return of an actively-managed portfolio to three different sources, without reference to any index or benchmark.

The success of the tactical component of an actively-managed portfolio should be judged entirely by the tactical return, as the strategic return and diversification return accrue regardless of the variation of the asset weights. The tactical return is independent of the total return of the portfolio. It can be positive even if the total return of the portfolio is negative, and vice versa. It is an absolute measure of success of the tactical trading decisions of the portfolio manager.

Strategic Return and Diversification Return

We begin by reviewing the strategic return and the diversification return of a rebalanced portfolio. More details may be found in Booth and Fama [1992], Bernstein and Wilkinson [1997], Erb and Harvey [2006], and Willenbrock [2011]. The reader that is only interested in the tactical return can skip this section.

The simple return of a portfolio, r_p, is the weighted average of the simple returns of the assets, r_i,

$$r_p = \sum_i w_i r_i \tag{1}$$

where the weights of the assets, w_i, satisfy $\sum_i w_i = 1$. The average of this equation over all holding periods is

$$\bar{r}_p = \sum_i w_i \, \bar{r}_i \tag{2}$$

where $\bar{r}_p$ and $\bar{r}_i$ are the arithmetic average returns of the portfolio and the assets, respectively. Since the weights, w_i, are constant, they factor from the average on the right-hand side of Eq. (2).

The actual growth of the portfolio and the assets is measured by the geometric average return, g, rather than the arithmetic average return, $\bar{r}$. The two are related by the approximate formula

$$g \approx \bar{r} - \frac{1}{2}\sigma^2 \tag{3}$$

where σ^2 is the variance of the simple returns. Using this equation on both sides of Eq. (2) gives

$$g_p + \frac{1}{2}\sigma_p^2 \approx \sum_i w_i \left(g_i + \frac{1}{2}\sigma_i^2\right) \ . \tag{4}$$

where g_p and g_i are the geometric average returns of the portfolio and the assets, respectively, and σ_p^2 and σ_i^2 are the variances of the portfolio and the assets, respectively. Hence

$$g_p \approx \sum_i w_i g_i + \frac{1}{2}\sum_i w_i \left({\sigma_i}^2 - {\sigma_p}^2\right) \tag{5}$$

The first term on the right-hand side of Eq. (5) is the strategic return,

$$\text{Strategic return} \equiv \sum_i w_i g_i \tag{6}$$

The second term on the right-hand side of Eq. (5) is an approximate formula for the diversification return, which can be defined precisely as the difference between the portfolio geometric average return and the strategic return:

$$\text{Diversification return} \equiv g_p - \sum_i w_i g_i \tag{7}$$

$$\approx \frac{1}{2}\sum_i w_i\left({\sigma_i}^2 - {\sigma_p}^2\right) \tag{8}$$

The strategic return depends only on the asset weights and their geometric average returns. It is the return a rebalanced portfolio would earn if the assets had zero volatility. The diversification return depends on the volatility of the assets, and is earned by the selling of assets that have appreciated in value and the buying of assets that have declined in value, relative to the value these assets would have had if they had zero volatility. The diversification return is positive, so it always adds an incremental return to the strategic return.

Diversification return is a useful concept because it explains how a rebalanced portfolio of risky assets can have a return greater than that expected from naively summing the returns of the assets [the strategic return, Eq. (6)]. Furthermore, the diversification return is earned while maintaining a constant risk profile via rebalancing. In that sense it is the "free dessert" that goes along with the "free lunch" of risk reduction inherent in a diversified portfolio.

The diversification return of a portfolio can be quite significant. Many examples may be found in Booth and Fama [1992], Bernstein and Wilkinson [1997], Erb and Harvey [2006], and Willenbrock [2011]. The concept is generalized to portfolios containing short and leveraged assets in Qian [2012]. Chambers and Zdanowicz [2014] show that the concept does not apply to expected returns, in contrast to realized returns.

In the following section we generalize the concepts of strategic return and diversification return to an actively-managed portfolio, and introduce the concept of tactical return.

Tactical Return

The simple return of a portfolio, r_p, is the weighted average of the simple returns of the assets, r_i,

$$r_p = \sum_i w_i r_i \tag{9}$$

where the weights of the assets, w_i, satisfy $\sum_i w_i = 1$. The average of this equation over all holding periods is

$$\bar{r}_p = \sum_i \overline{w_i r_i} \tag{10}$$

where $\bar{r}_p$ is the arithmetic average return of the portfolio. In an actively-managed portfolio, the asset weights, w_i, vary, so they cannot be factored out of the average on the right-hand side of Eq. (10) as they were in Eq. (2). Instead, we use the relation between the average of the product of two variables and the product of the averages,

$$\overline{xy} = \bar{x}\bar{y} + cov(x, y) \tag{11}$$

where the second term on the right-hand side of Eq. (11) is the covariance of the two variables. Applied to Eq. (10), we find

$$\bar{r}_p = \sum_i \bar{w}_i \bar{r}_i + \sum_i cov(w_i, r_i) \tag{12}$$

We now follow exactly the same steps as in the previous section [Eqs. (3)-(5)] to arrive at

$$g_p \approx \sum_i cov(w_i, r_i) + \sum_i \overline{w}_i g_i + \frac{1}{2}\sum_i \overline{w}_i \left(\sigma_i{}^2 - \sigma_p{}^2\right) \tag{13}$$

where the last two terms on the right-hand side of Eq. (13) are identical to Eq. (5), but with the fixed weights replaced with average weights. The first term is new and is thus associated with the tactical trading decisions of the portfolio manager. We define the tactical return as

$$\text{Tactical return } \equiv \sum_i cov(w_i, r_i) \tag{14}$$

and following the logic of the previous section, we define

$$\text{Strategic return} \equiv \sum_i \overline{w}_i\, g_i \tag{15}$$

$$\text{Diversification return} \equiv g_p - \sum_i cov(w_i, r_i) - \sum_i \overline{w}_i\, g_i \tag{16}$$

$$\approx \frac{1}{2}\sum_i \overline{w}_i\left(\sigma_i{}^2 - \sigma_p{}^2\right) \tag{17}$$

The formula for the tactical return, Eq. (14), makes intuitive sense. The covariance receives a positive contribution from any holding period in which the weight of an asset is greater than its average weight while the return of that asset is greater than its average return. But the covariance also receives a positive contribution whenever an asset's weight is less than average while its return is less than average. On the other hand, the covariance receives a negative contribution whenever the asset's weight is less than average while its return is greater than average, and vice versa.

The formulae for strategic return [Eq. (15)] and diversification return [Eq. (17)] also make intuitive sense. In a portfolio with varying asset weights, it is their average values that naturally replace the fixed weights of the previous section. The tactical return is accrued by varying the weights around their average values.

An example of an actively-managed portfolio is given in Table 1. This is an example of a tactical asset allocation portfolio. Shown are the total annual returns of the S&P 500 index and the Barclays Capital US Long Treasury Bond index for the volatile decade spanning 2000 – 2009. Two portfolios are shown. One portfolio rebalances to a 50/50 mix of the two assets annually. The other portfolio varies the weights to 60/40 or 40/60 annually. We imagine that the portfolio manager was usually successful in anticipating which asset would perform best in the coming year, so we overweight the Treasury Bond index in the five years in which it outperformed the S&P 500 index the most (2000-2002, 2007-2008, indicated by an asterisk). Thus the average weights of the assets is 50/50, the same as the 50/50 portfolio. The geometric average return of the tactical portfolio is 6.79%, significantly greater than the 4.44% of the 50/50 portfolio. Almost all the difference can be credited to the tactical return, which is 2.31%. The strategic return, 3.32%, is identical to that of the 50/50 portfolio, since the two portfolios have the same average weights. The diversification return is 1.16%, slightly greater than the 1.12% of the 50/50 portfolio.

Because the diversification return is significant, it would be misleading to compare the return of the tactical portfolio to only the strategic return in order to judge the success of the tactical asset allocation decisions. The difference between the portfolio geometric average return, 6.79%, and the strategic return, 3.32%, is 3.47%. Not all of this is due to astute tactical decisions; roughly one third (1.16%) is the diversification return, and the other two thirds (2.31%) is the tactical return. Only the tactical return should be credited to the portfolio manager's tactical decisions.

Nonperiodic Trading

The formulae for tactical return, strategic return, and diversification return are straightforward to apply if the trading of assets is done periodically, as in the example in Table 1. However, no active manager trades on a periodic schedule. We need to consider how to apply the formulae for tactical return, strategic return, and diversification return to a portfolio with nonperiodic trading. We did not assume that the trading was done periodically when we derived these quantities, so we need only apply the formulae as they are written, and follow a procedure that will yield the annualized returns. We will do this by example. We first discuss a strategic asset allocation, where the weights, w_i, are constant. We then consider a tactical asset allocation, where the weights vary.

We show in Table 2 an example of a strategic asset allocation with nonperiodic trading. We have used the returns from Table 1, but modified the trading intervals. Rather than rebalancing annually, the 50/50 portfolio is rebalanced after 3 years, then after 1, 3, 2, and 1 years. The simple returns (which are not annualized) were calculated from the simple returns of Table 1.

By changing the rebalancing schedule, we have increased the geometric average return of the 50/50 portfolio to 5.08%, compared with 4.44% for the annually-rebalanced portfolio of Table 1. We now discuss how to separate this return into a strategic return and a diversification return.

We give in Table 2 the arithmetic average return, $\bar{r}$, the geometric average return, g, and the standard deviation, σ, for the nonperiodic portfolio. Since there are five holding periods over a ten year span, all of these quantities correspond to a biannual average, rather than an annual average. The geometric average return is 10.42% (biannual). The strategic return, given by Eq. (6), is 6.93% (biannual). The diversification return, given by Eq. (7), is 3.49% (biannual). These results tell us that 66.5% of the portfolio geometric average return is a strategic return, and the remainder is a diversification return. Compare this with the annually-rebalanced portfolio of Table 1, where the strategic return accounted for 75% of the geometric average return. The diversification return has increased in importance because, with an average holding period of two years, the assets are allowed to drift further from their target allocations between rebalancings.

By convention, returns are usually annualized. One cannot simply annualize all three of the returns (geometric average return, strategic return, diversification return), because they would not sum together properly, due to the nonlinear effects of compounding. Instead we annualize the geometric average return, and split it into a strategic return and diversification return such as to maintain the percentage of the return from each that we found biannually. The annualized returns are listed in the bottom section of Table 2.

Now consider a tactical asset allocation portfolio. Following the example in the previous section, we overweight the Treasury Bond index 60/40 in the two holding periods that it most outperforms the S&P 500 index (2000-2002, 2007-2008, indicated by an asterisk). In the other holding periods, we overweight the S&P 500 index 60/40. This boosts the portfolio geometric average return to 7.38%, compared to 5.08% in the 50/50 portfolio. This corresponds to a geometric average return of 15.31% (biannual), as shown in Table 2.

The tactical return, calculated from Eq. (14), is 5.02% (biannual). To determine the strategic return, we need the average weights of the assets in the portfolio. Since the S&P 500 index is overweighted 60/40 in three of the five holding periods, it has an average weight of 52%, while the Treasury Bond index has an average weight of 48%. Using these in Eq. (15), we find a strategic return of 6.58% (biannual). The diversification return, given by Eq. (16), is 3.71% (biannual).

We can annualize these returns in a manner similar to that described above. We first annualize the geometric average return of the portfolio, and then assign the annualized tactical return, strategic return, and diversification return *pro rata*. These annualized returns are listed in the bottom section of Table 2. The tactical return (2.42%) is close to that of the annually-traded tactical portfolio of Table 1 (2.31%) because the tactical decisions are identical. The strategic returns of all four portfolios are quite close because the average weights of the assets are nearly the same. The diversification returns of the two nonperiodic portfolios are similar, and greater than those of the annually-traded portfolios because the assets are allowed to drift further from their allocations, as mentioned previously.

In an actively-managed portfolio the trading generally occurs much more frequently than annually, in contrast to the example provided here. Nevertheless the methodology is identical, with each holding period corresponding to the interval between successive trades.

Conclusions

We have provided an objective measure of the success of the tactical decisions made by the portfolio manager of an actively-managed portfolio. We have shown how to decompose a portfolio's geometric average return into three well-defined parts: tactical return, strategic return, and diversification return. Only the tactical return should be credited to the portfolio manager's tactical decisions. We applied these results to both strategic and tactical portfolios, with trading done either annually or nonperiodically, and showed that the decomposition of the portfolio geometric average return behaved sensibly in all portfolios. The methodology is straightforward and can be automated and applied to an arbitrarily complicated portfolio.

Note added: A similar decomposition, also making use of Eq. (12), has been performed by Hsu, Kalesnik, and Myers [2010] on a traditional multiperiod Brinson attribution analysis which, unlike our analysis, compares the portfolio to a benchmark portfolio. Their static allocation effect is the analogue of our strategic return, and their dynamic allocation effect is the analogue of our tactical return.

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Table 1: a) A strategic portfolio with 50% invested in the S&P 500 Index Total Return and 50% invested in the Barclays Capital US Long Treasury Index on January 1, 2000. b) A tactical portfolio with asset weightings 60/40, with the Treasury Bond index overweighted in the years marked with an asterisk and the S&P 500 index overweighted in the other years. Return data from the Vanguard Group.

Year Ended	S&P 500 TR (%)	Long Treasuries (%)	50/50 Portfolio (%)	60/40 Portfolio (%)
2000*	(9.10)	20.27	5.58	8.52
2001*	(11.89)	4.21	(3.84)	(2.23)
2002*	(22.10)	16.79	(2.66)	1.23
2003	28.68	2.48	15.58	18.20
2004	10.88	7.70	9.29	9.61
2005	4.91	6.50	5.70	5.55
2006	15.79	1.85	8.82	10.21
2007*	5.49	9.81	7.65	8.08
2008*	(37.00)	24.03	(6.48)	(0.38)
2009	26.46	(12.92)	6.77	10.71
$\bar{r}$ (%)	1.21	8.07	4.64	6.95
g (%)	(0.95)	7.59	4.44	6.79
σ (%)	20.03	10.05	6.51	5.79

	50/50 Portfolio (%)	60/40 Portfolio (%)
Geometric average return (%)	4.44	6.79
Tactical return (%) [Eq. (14)]	–	2.31
Strategic return (%) [Eq. (6,15)]	3.32	3.32
Diversification return (%) [Eq. (7,16)]	1.12	1.16
Diversification return approx. (%) [Eq. (8,17)]	1.04	1.09

Table 2: Same as Table 1, but with nonperiodic trading. The simple returns are calculated from the simple returns of Table 1. The portfolio returns are given both biannually and annually.

Year Ended	S&P 500 TR (%)	Long Treasuries (%)	50/50 Portfolio (%)	60/40 Portfolio (%)
2000-2002*	(37.61)	46.38	4.38	12.78
2003	28.68	2.48	15.58	18.20
2004-2006	34.69	16.82	25.76	27.54
2007-2008*	(33.54)	36.20	1.33	8.30
2009	26.46	(12.92)	6.77	10.71
$\bar{r}$ (%)	3.74	17.79	10.76	15.51
g (%)	(1.89)	15.76	10.42	15.31
σ (%)	32.23	21.61	8.87	6.85

	50/50 Portfolio (%)	60/40 Portfolio (%)
<u>Biannual</u>		
Geometric average return (%)	10.42	15.31
Tactical return (%) [Eq. (14)]	–	5.02
Strategic return (%) [Eq. (6,15)]	6.93	6.58
Diversification return (%) [Eq. (7,16)]	3.49	3.71
Diversification return approx. (%) [Eq. (8,17)]	3.37	3.59
<u>Annual</u>		
Geometric average return (%)	5.08	7.38
Tactical return (%)	–	2.42
Strategic return (%)	3.38	3.17
Diversification return (%)	1.70	1.79