Asset Allocation Math, Methods and Mistakes

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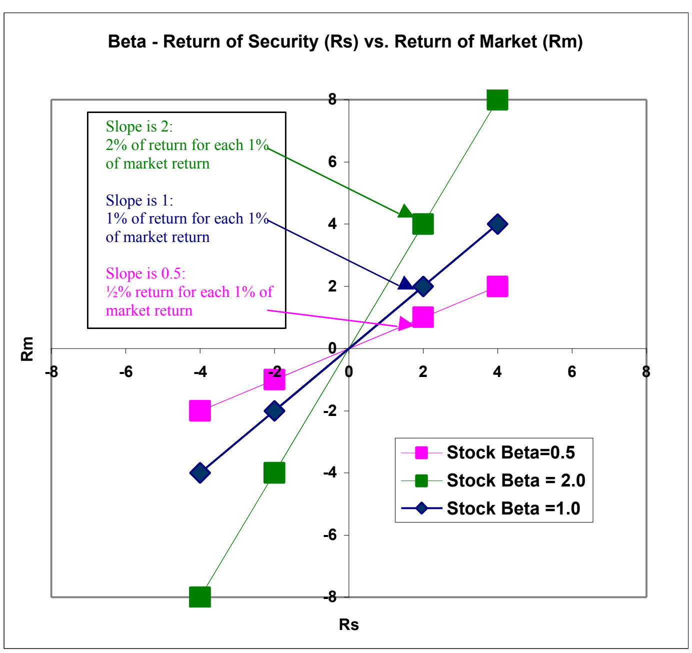
Over the last ten years, some very basic premises about asset allocation have been forgotten. The roots of asset allocation theory started with a piece of work called "The Capital Asset Pricing Model" (CAPM- beta as a measure of risk) and evolved into "Modern Portfolio Theory" (MPT- Standard deviation as a measure of risk). Other works, particularly the Brinson, Hood and Beebower studies of 1986 and 1991, have been mistakenly understood as "proof" that 90% or more of investment returns are "due to" asset allocation (ignoring how the study defined asset allocation). This misinterpretation and how it is being applied in practice resulted in contradictory conclusions from the very work it was attempting to validate. More recently, returns-based style analysis has been a further attempt to explain investment performance and is currently in vogue. This paper will revisit some of these works, expose what can rationally be concluded from them, and disclose risks in misunderstanding their logical application.

Capital Asset Pricing Model (CAPM)

CAPM is a very basic formula designed in an attempt to explain some perceived contradictions in the behavior of individual stocks and the overall stock market. It is one of the earlier works trying to explain the rationale for diversification. Its focus is on diversification of stock portfolios to produce certain risk and reward characteristics through the use of risk free investments and leverage--unlike MPT, which threw bonds into the mix. A primary premise of CAPM is to explain why investors diversify stock portfolios.

The CAPM theory revolves around the idea that there are basically two kinds of risk: diversifiable risk (non-systematic or "event risk") and non-diversifiable risk (systematic or "market risk"). The premise is that investors would not subject themselves to a risk that could be diversified away and that there should not be an expectation for additional reward in accepting a diversifiable risk. While theoretically we believe this makes sense, the actual behavior of investors is in contradiction to this theory; otherwise investors would not expose themselves to "concentration risk."

Beta is an important part of this theory and is assumed to be how investors measure risk. Mathematically, beta measures the relative sensitivity of a security to overall market movements. It is calculated by running a linear regression of the returns of a security versus a reasonable approximation of a "diversified market basket" of stocks in general. The slope of the regression line indicates the beta of the security (see *Exhibit 1*).



The beta (ß) of a security is based on the slope of a linear regression line when plotting the returns of the security (Rs) versus the returns of the market (Rm).

A stock with a beta of 2.0 has a slope that infers that when the market produces a return of 4% the security produces a return of 8%.

A stock with a beta of 1.0 has a slope that infers that when the market produces a return of 4% the security also produces a return of 4%.

A stock with a beta of 0.5 has a slope that infers that when the market produces a return of 4% the security produces a return of 2%.

CAPM assumes that beta is how investors measure the risk they are willing to assume and that investors are compensated for accepting this risk. If we accept this, beta could therefore be used to forecast the return of a security by applying the CAPM formula. This was the initial risk versus return trade off scenario that was assumed to be used in constructing diversified portfolios. These portfolios would be expected to produce a return the investor is seeking for the risk (beta) they are willing to assume.

Exhibit 2 - The Capital Asset Pricing Model Formula

CAPM Formula:

$$R_S=R_F + (ß_S \times (R_M - R_F))$$

Where:
R_S = Expected Return of a Security
R_F = Risk Free Rate of Return
$ß_S$ = Beta of a Security
R_M = Expected Return of The Market

In this formula (see *Exhibit 2*), it is assumed that the beta of a security determines its expected return. The "Equity Risk Premium" (the return of the market less the risk free rate of return) is multiplied by the beta of a security and added to the risk free rate of return to calculate a security's expected return.

CAPM makes a few other assumptions, perhaps the most egregious one being that an investor can borrow and lend at the risk free rate of return. A simple check of the interest rate charged on margin accounts versus the interest rate earned on money market accounts exemplifies this erroneous assumption in the theory.

However, if we accept the premises of CAPM, we see the first evidence of why investors would want to diversify. For example, let's say I was a risk averse investor and I was only willing to accept a beta of 0.5. One easy way of achieving that beta would be to simply own a single stock that had a beta of 0.5. The return I should expect would be equal to the risk free rate of return plus ½ of the equity risk premium. We can easily apply the formula to learn the expected return of the security (see *Exhibit 3*)

Exhibit 3 - Calculating the expected return of a security with a beta of 0.5

Assumptions:
Risk Free Rate of Return: 4%
Expected Return of the Market: 12%
Beta of Security (Risk Tolerance): 0.5

CAPM Formula:

$$R_S=R_F + (ß_S \times (R_M - R_F))$$
$$R_S=4\% + (0.5 \times (12\% - 4\%))$$
$$R_S=4\% + (0.5 \times (8\%))$$
$$R_S=4\% + 4\%$$
$$R_S=8\%$$

If my assumptions are correct I should expect an 8% return on a security with a beta of 0.5. However, if I owned a single stock to produce this return I am taking a lot of risk that has to do with "events" that could influence that security that would not materially affect the overall market. What happens if the company's CEO dies in a plane crash? What if one of their plants experiences a catastrophic explosion, fire, or is destroyed by a tornado? What if they experience serious disputes with labor that result in the loss of confidence of their customers? What if they make a mistake in the design of a new product that creates significant product liability claims?

CAPM assumes that risks of this sort are not rewarded because investors have an alternative means of achieving the return of that security without assuming these risks (which could be diversified away). The assumption is that instead of owning a single security, I can "manufacture" a portfolio that will produce the same beta, and therefore the same expected return, without subjecting me to these "event" risks.

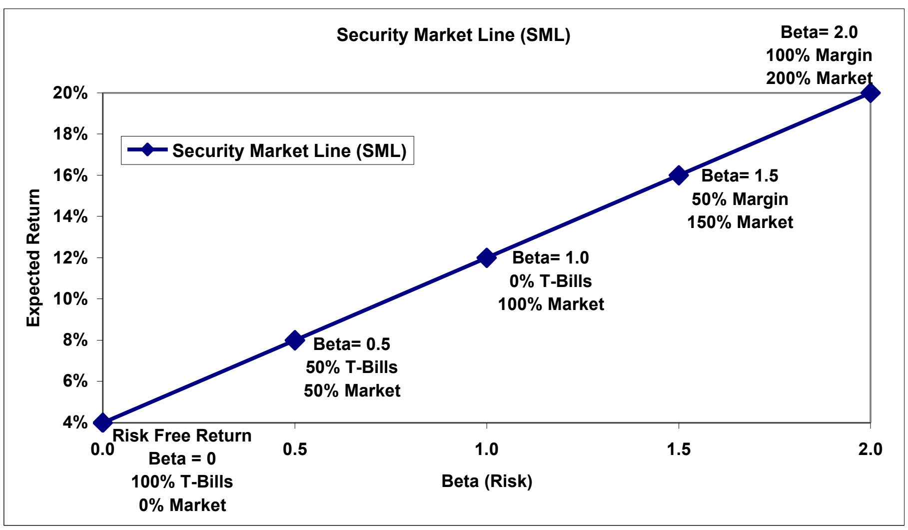
Instead of putting all of my money in a single stock with a beta of 0.5, I can produce the exact same risk and return characteristics by investing half my money in risk-less assets and the other half in a diversified "market portfolio." In fact, according to CAPM, investors can manufacture a portfolio with any risk (beta) and return characteristics they desire by simply borrowing (leverage or margin) or lending (investing in T-Bills) at the risk free rate of return, complemented by a diversified portfolio of stocks (see *Exhibit 4*).

Exhibit 4 - Borrowing & Lending at the Risk Free Rate of Return to Avoid Un-rewarded Diversifiable Risk

Low Beta Stock Beta = .5	Market Beta Stock Beta = 1.0	High Beta Stock Beta = 2.0	
R=4+((.5)*(12%-4%)) R= 4+(.5*8%) R= 8% OR	R=4+((1.0)*(12%-4%)) R= 4+(1*8%) R= 12% OR	R=4+((2.0)*(12%-4%)) R= 4+(2*8%) R= 20% OR	CAPM – The Expected Return of a security based on its beta
Diversified Portfolio: R=Portfolio Weight*Return R= (50%*4%)+(50%*12%) R=2%+6% R=8% 50% Invested in T-Bills 50% Invested in "Market"	Diversified Portfolio: R=Portfolio Weight*Return R= (0%*4%)+(100%*12%) R=0%+12% R=12% 0% Invested in T-Bills 100% Invested in "Market"	Diversified Portfolio: R=Portfolio Weight*Return R= (-100%*4%)+(200%*12%) R=-4%+24% R=20% 100% Borrowed at Risk Free Rate 200% Invested in "Market"	Equivalent "Market Portfolios" constructed to produce identical returns without assuming uncompensated "event" risk

These diversified portfolios can be plotted, thus creating the "Security Market Line" (SML) where investors simply identify their tolerance for risk (beta) and balance their ownership of a diversified market portfolio with risk-less assets or debt (see *Exhibit 5*).

Exhibit 5 - The Security Market Line (SML)



CAPM has never been proven and in fact has come under some significant criticism based on some contradictory empirical data. Beta has not been proven as a means of forecasting returns, and clearly the assumption that an investor can both borrow and lend at the risk free rate of return is absurd. For all of its flaws though, there is something to the rationale that risk one doesn't need to assume (risk that could be diversified away) would not be rewarded with extra return.

So, what can we learn from CAPM and what problems of CAPM are addressed by the later works? We believe that it intuitively makes sense to diversify stock portfolios. There are too many uncertainties and far too much variance in portfolios made up of one or two stocks to assume that those "bets" would be compensated. The uncertainty and volatility in concentrated portfolios leaves us with a "crap shoot" that cannot be assumed to produce a higher reward. (Watch for our upcoming paper, "The Concentration Crisis" to learn more about this risk.)

The amount of this uncertainty creates a reasonably equal risk of both really huge rewards AND really terrible results, therefore increasing the *range* of potential results *but not increasing the expected result.* It is important to note that this risk is based on the weighting of stocks in a portfolio and not the total number of stocks in a portfolio. A portfolio made up of 500 stocks in equal proportions, or weighted by market capitalization, will be far less volatile than a portfolio that has 499 stocks in equal proportion making up half the portfolio and the other half in one stock of average volatility. This is true even if the betas of the two portfolios are the same. The median stock has more than twice the standard deviation (NOT beta) as the market so my portfolio's volatility is determined by the weighting of positions, their correlation to one another and their volatility, not solely on the number of positions. We believe the conclusion of CAPM, that it makes sense to diversify, is a valid conclusion.

There has been much work done to measure how many stocks one should own (again, assuming that no position is significantly over weighted) to materially eliminate this risk. It is, of course, dependent on a number of factors such as industry and sector diversification, among other characteristics. The consensus ranges from 20-35 stocks generally, and we will not attempt to identify the "right" number as the diminishing materiality is significant.

The question of whether investors use beta as their sole risk measure, as CAPM assumes, we believe is an erroneous conclusion of CAPM. MPT favors standard deviation as a risk measure, since it measures the unpredictability of portfolio returns regardless of the market performance; whereas beta only measures the sensitivity to the market. We do not believe that investors' only concern is their performance relative to the markets.

We also believe there is enough contradictory data and valid studies to conclude that beta is not necessarily a predictor of returns, or at least that there is not sufficient evidence to draw the conclusion that beta is a predictor of returns. Further evidence for this conclusion can be drawn from the evidence of how many professional managers can consistently out-smart the market. Our ability to choose winning stocks and how many professionals are unable to consistently do so, demonstrates in my mind that CAPM's conclusion that beta predicts expected return is erroneous. If all it took to outperform the market was to have a higher beta than the market, many more managers would beat the market.

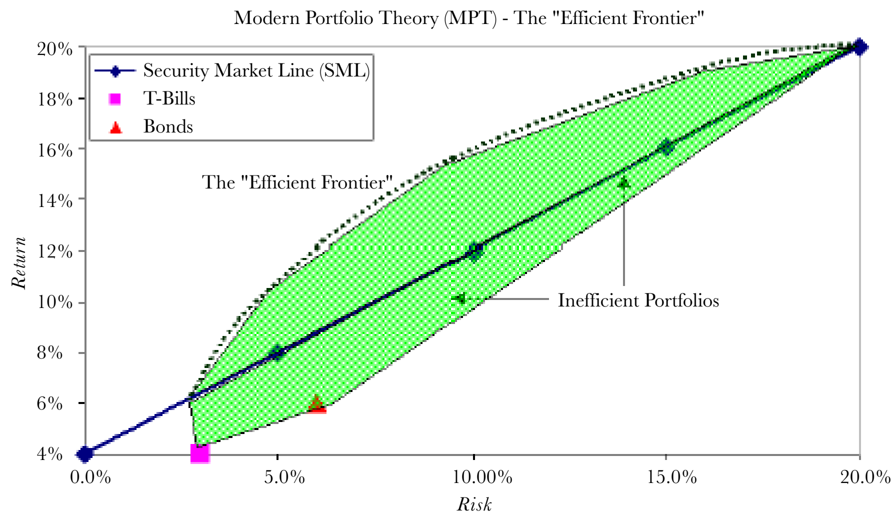
Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT) concludes that investors measure their tolerance for risk not by beta relative to the market but instead by standard deviation. Standard deviation is significantly different than beta. While they both measure volatility, beta measures volatility relative to the market and standard deviation measures volatility relative to its mean return. The premise of MPT is that investors are willing to assume a certain amount of unpredictability in their investment returns and that this is not necessarily relative to market movements. So, while beta measures how sensitive a portfolio is to market movements (extreme or otherwise), standard deviation measures the extent and frequency of the variance in a portfolio's returns, or how unpredictable returns would be.

MPT deals with portfolios under the assumption that CAPM is correct, in that investors will diversify away "event" risks they are not paid to assume. Betas of portfolios could be calculated just as the beta of a stock could be calculated, by running a linear regression of the portfolio returns versus market returns. This becomes more difficult, however, as you throw another asset into the portfolio mix like bonds. CAPM doesn't really deal with bonds…there are simply stocks and risk free assets in the CAPM model. In CAPM, it is reasonably easy to identify a "market portfolio" of stocks, since once one creates a fairly broadly diversified "market basket"; the returns of any other similarly diversified "market basket" become, for all intents and statistical purposes, similar.

But, how does one identify a "market basket" that includes not only ownership in companies and risk free investments like CAPM, but now also includes debt which has its own risks that are separate from the risks of the overall equity market? By adding this third dimension to our risk/return chart we see that instead of a single security market line representing all potential equity portfolios blended with risk free assets, we now have an "area" based on the characteristics of portfolios blended in varying proportions to these three primary assets (see *Exhibit 6*).

Exhibit 6 - Potential portfolios represent an "Area" and those that produce the highest return per unit of risk represent the "efficient frontier."



An important statistic that is inherent in MPT is that in addition to risk and return, one needs to know the correlation of the assets to one another. This is actually a problem with beta in CAPM as well. You may recall that in Exhibit 1, by using beta one can infer the return of a security given a specific market return. The individual returns in any one period are not necessarily proportionate to the beta as the linear regression line inferred. The regression line in fact is the "best fit" of what one could draw. Often these "best fits" are really not very good fits at all. To measure this, a statistic known as r-squared can be used to measure how well or how poorly the linear regression line fit the pattern of returns. If the line were a perfect fit, the r-squared would be equal to 1.00 and a perfectly straight line could be drawn through the "dots." The lower the r-squared, the poorer the "fit" and therefore the more uncertainty that beta would forecast the relative return of the security to the market in any one period.

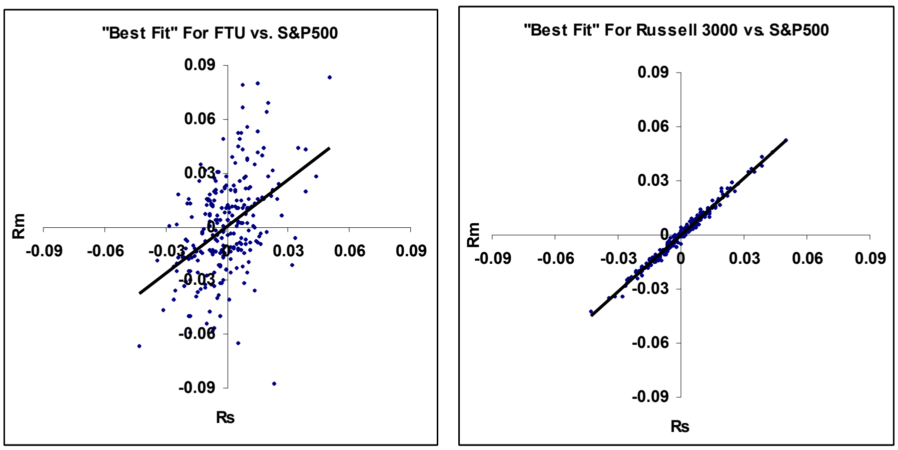
Exhibit 7 compares a stock (FTU - First Union) and a broad index (Russell 3000), both of which have a beta of approximately 1.0, to the S&P 500.

Exhibit 7- Comparison of a Stock and an Index with a beta of approximately 1.0 to the S&P 500

S&P500
Beta= 1.0
R-Squared= 1.00
Standard Deviation= 22.05%
Correlation= 1.00

FTU (First Union)to S&P500
R-Squared: 20.22%
Standard Deviation: 42.74%
Correlation: .4496

Russell 3000 to S&P500
R-Squared: 98.59%
Standard Deviation: 23.33%
Correlation: .9929



Where Things Got Off Track

The famous Brinson, Hood & Beebower study that has been misinterpreted as stating that 90% or more of return was "due to" asset allocation is based on r-squared. More accurately stated, the study said that more than 90% of the variance in portfolio returns could be explained by asset allocation to stocks, bonds and T-bills. We will not be addressing other conclusions of the study other than mentioning that average returns were negatively impacted by security selection and timing decisions. Much of the popularity of this work was based on this return attribution information by advocates of indexing.

The study took 91 pension funds and calculated the r-squared of the returns of these large, diversified pension funds relative to the return one would achieve in market indices of similar proportion. There should absolutely be no surprise that the variance of returns of these broadly diversified pension funds "closely fit" the returns of unmanaged indices that are also broadly diversified. In fact, any portfolio that is reasonably well-diversified will by definition have both a relatively high r-squared and correlation.

It amazes me how many conclusions have been drawn from this study, even by industry experts. This study has been revered as evidence that asset allocation "works" and has been further extrapolated into the justification for returns-based style analysis and our expansion beyond the already difficult-to-predict relationship of risk, returns, and correlations of "macro" asset classes like stocks, bonds, and cash into "sub-classes" like mid-cap value, large-cap growth, etc.

The Brinson study in fact defined asset classes as T-bills, government bonds and stocks. It is bewildering to me that a study that concluded that over 90% of the variance in returns could be explained by their stock, bond and T-bill allocations is being used to justify our feeble attempts to optimize style and market capitalization. To the contrary, since the study did not attempt to measure these attributes, one could reasonably conclude that style and capitalization bets account for less than 10% of the variance in returns since 90% could be explained through the broad asset classes. Yet, advisors that attempt to practice the current allocation vogue use the study to justify something it did not measure and actually contradicts!

Before we put too much weight in the study, draw potentially erroneous conclusions, and expand its meaning beyond what might be rational to conclude, it might be helpful to think about what conditions would be needed to have the results of the study be a lower r-squared than the study calculated? As you can see in *Exhibit 7*, the r-squared or "fit" of the regression line of a single stock relative to the market can produce a low r-squared. How many large pension funds do you know of that invest all of their assets in a single stock? If they did, the Brinson study would have shown asset allocation as a much lower "source" of investment return variance. This is, of course, in direct conflict with the one rational conclusion we can draw from CAPM, as well as prudent expert rules, etc. Pension funds do not make that kind of bet. They diversify.

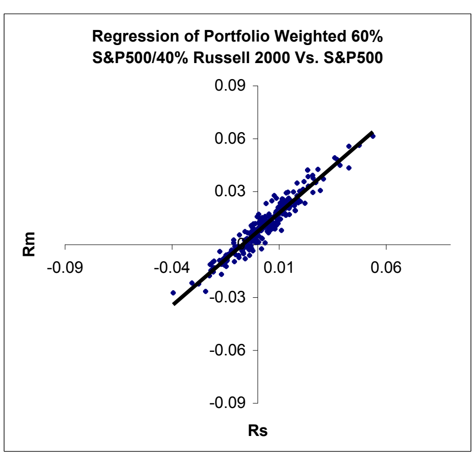
Of course, pension funds could have had a lower r-squared if they radically attempted to time the market. For a pension fund (and most investors) this is risky as well, since being wrong on such a "bet" has too high of a probability of occurrence relative to the reward of the exceptional performance you would achieve if you were right. This is not in the nature of how large pension funds manage their portfolios.

Having served on the investment advisory committee of the $30 billion Virginia Retirement System for several years, I can tell you from practical experience that large pension funds do not make these sorts of bets. In general, pension funds diversify their portfolios and do not make extreme market timing bets. This is not only something that is supported in the wisdom of CAPM's conclusions about diversifying to avoid uncompensated risks, but is in fact the nature of the behavior of large pension funds.

What kind of bets could be made to produce a lower r-squared of the portfolio? What if my benchmark were the S&P 500? How much of a "bet" could I make against the index and still produce a high r-squared? What if I took 40% of my portfolio and invested it in small cap stocks like the Russell 2000? This would be a HUGE bet against my benchmark. Both the Russell 2000 and S&P 500 are market capitalization weighted indices, and since the Russell 2000 is the smallest 2000 of the largest 3000 stocks, there effectively is little material overlap between this index and the S&P 500. To any pension fund, this would be nothing short of an "insane" bet.

However, if we run the regression and calculate the results, we find that the r-squared is .92! We can see how closely this "wacky" portfolio bet still correlates to the benchmark in *Exhibit 8*. (This was calculated using daily returns for the last year for comparison to the beta calculations as in the FTU and Russell 3000 calculations. Even using annual returns back to 1926 though, the r-squared of the 60% large cap and 40% small cap portfolio using annual Ibbotson data has an r-squared of .89)

Exhibit 8 - A portfolio weighted 40% small cap has an r-squared relative to the S&P 500 of 92.



With such a fundemental portfolio weighting difference of market capitalization we still produce an r-squared of .92 and a correlation of .95

Is it rational to conclude that therefore we must further define asset classes into style and other market capitalization classes?

To me, the only thing really "proved" by the Brinson, Hood & Beebower study is that pension funds are diversified. I really didn't need a study to tell me that. The only way the r-squared *could have been low* was if pension funds ignored the prudence of diversification and concentrated their assets in non-diversified portfolios or radically timed the market. This is a "no-no" in CAPM, MPT and ERISA's "prudent expert" rule.

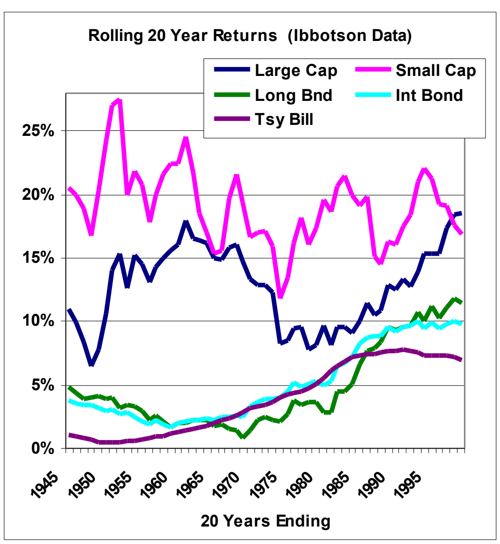
The results of the study have been misinterpreted to mean that the asset classes you hold, regardless of whether you are diversified or not, are the driving force of your investment results. At least, this is how it has been applied. It also has been held out as the "proof" that Modern Portfolio Theory or MPT "works." I do not know the motivation of Brinson & Beebower, but if they are reasonable statisticians they would definitively want to avoid these conclusions.

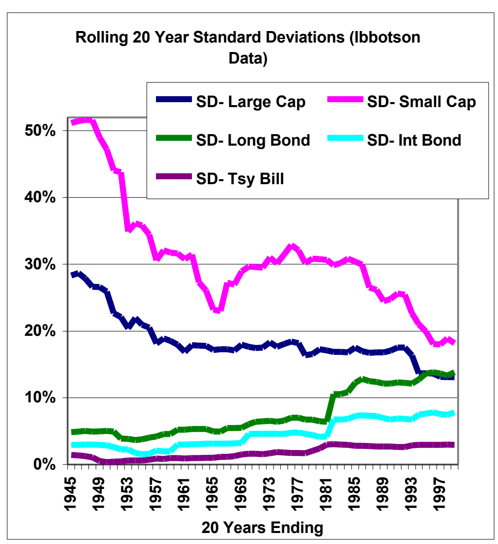
We have been on record as being critical of MPT in our papers Modern Portfolio Reality and The Use of Monte Carlo in Modern Portfolio Reality. This has much less to do with the fundamentals of MPT and more to do with the way MPT is being applied, or should we say, misapplied. Just as CAPM had a significant problem with the "fit" of beta for any single security, MPT has a similar problem when dealing with what we call "sub-classes."

To create an efficient frontier, a mean variance optimizer is applied to the risk, return, and correlations of asset classes. Lately, "stochastic optimizers" running Monte Carlo simulations have been used for "better" asset allocation optimizations. For the optimization to be valid and "prudent" one needs to have a high confidence level in ALL of the inputs to the optimizer...that is...the risk, return, and correlations between ALL of the asset classes. One slip-up on our estimates and the resulting efficient frontier can put us squarely in inefficient territory. Having studied this for the past sixteen or so years, I'm almost of the opinion that the way most optimizers are run in practice is nearly a contrary indicator. This has to do with the inputs, not the forumulas, and the reality of our inability to forecast risk, return, and correlations.

Many optimizers use the historical returns for the last 10, 20 or 30 years as the "expected return." Sounds reasonable enough...but if we "check our premises" we can see that this may be an erroneous assumption worthy of being questioned for both returns and standard deviations.

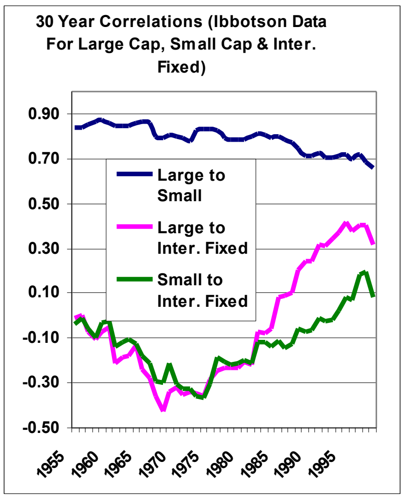
Exhibit 9 - Rolling 20-Year Mean Returns and Standard Deviations (Ibbotson Data)





Imagine the differences in your optimizer's suggested portfolio weights if you presume a 10% return advantage to small cap stocks over large cap stocks as the 20-year returns inferred in the late 1970's and early eighties. The standard deviations are also difficult to predict even with long periods of time, although somewhat less so. Finally, as shown in *Exhibit 10*, correlations between bonds and stocks appear highly unpredictable, but, as CAPM would infer, the correlations between diversified baskets of stocks are high.

Exhibit 10 - Rolling 30-Year Correlations (Ibbotson Data)



"Over Optimization"

Where things have really gotten off track has been the insistence on breaking asset classes into sub-classes by style, market capitalization, etc. The unpredictability of all of the inputs into our optimizers, even over long periods of time, has been ignored. We have attempted to take efficient portfolios of stocks, bonds and cash and make them even more efficient by breaking the unpredictable asset classes into even less predictable sub-classes.

This all being done under the pursuit of "efficiency" as was supposedly validated by the Brinson & Beebower study, which purports to find that over 90% of the investment return variance is explained by asset allocation.

The risk that you will produce inefficient portfolios INCREASES as you increase the number of "asset classes" for which you must forecast not only the risk and return, but also each asset class' correlation to the others. The results of the optimizer and your resulting portfolio's efficiency is based on the accuracy of inputs and NOT THE NUMBER OF INPUTS.

An efficient portfolio WILL NOT BE the result of my efforts if my inputs into the optimizer are materially mis-estimated. Maybe you are good enough to look at the data and forecast these relationships. Maybe you know what the correlations, standard deviations, and returns will be for mid-cap value stocks vs. large cap growth stocks.

As for other classes like real estate, foreign stocks and private equity, there was likely to be some allocation to these classes within the funds included in the Brinson study, but if there was, they also did not impact the results significantly. Allocations to these classes are generally not very extreme and most return variance would still be explained by stocks, bonds and T-bills. Further evidence of this could be inferred based on the timing of the two studies since the record of real estate after our inflationary cycle of the late '70s and early '80s caused pension funds to increase their allocation to real estate and at least in the late '80s, their foreign allocation as well, yet the second study still confirmed the high r-squared to the three major asset classes. Perhaps the study excluded funds that had allocations to anything other than stocks, bonds and cash. If so, I doubt whether the results would have been materially different.

We believe there are some rational criticisms of MPT as further explained in our "Modern Portfolio Reality" papers. We do not believe investors have a maximum "tolerance" for standard deviation that they can identify or even relate to. But even if they did, *we also believe it is erroneous to assume that once that maximum tolerance for risk was identified, they would then proceed to create portfolios designed to experience it!* This was a fundamental assumption of MPT. Instead, we believe that risk is something investors prefer to avoid as much as possible, but they begrudgingly accept as a preferable choice relative to the contrasting effects on their lifestyle for not accepting investment risk. If this makes sense to you, then one can only make the risk tolerance decision not based on the relative return, but instead relative to what it means to the likelihood of achieving investor's goals and the corresponding impact to things such as savings rates, retirement age, retirement income and estate goals. MPT ignored these real world decisions. Modern Portfolio

Reality is based upon them.

The other criticism of MPT we have is more in how it is being applied rather than the mathematics of the theory itself. Take our small cap “bet” for our theoretical pension fund with an S&P 500 investment policy. It is hard to imagine that someone in 1979, looking at a 9% small cap stock return premium and corresponding 14% higher standard deviation for the last twenty years, would forecast the relationship over the next twenty years to shift to small caps under-performing large caps by nearly 2% and their standard deviation being less than 2% higher than the 20 year standard deviation of large caps in 1979.

Exhibit 11: Twenty Year Risk and Returns – Small Cap Vs. Large Cap (Ibbotson Data)

	1979			**1999**		
	Risk	Return	Correlation	Risk	Return	Correlation
Small Cap Stocks	30.8%	17.4%	.78	18.1%	16.9%	.59
Large Cap Stocks	16.5%	8.1%		13.1%	18.6%	

How efficient a portfolio would you have had in the twenty years ending in 1999 if you were looking at the data in 1979 and “optimized” based on that data? Some compensating judgment may have helped but still would have likely placed you in inefficient territory. Our major criticism of MPT is how it is being applied. Advisors have abandoned judgment for theoretical precision. They have generally ignored that the mathematics of MPT are the only fact of MPT. They have ignored the fact that the mathematics ONLY work if the inputs to the formula are precise. The precision they have been applying is further refinement of the inputs; which produces the opposite effect of their objective. If we know that it is difficult to forecast even broad asset class relationships over long periods of time, why do we believe that the more relationships I’m unable to forecast will increase the accuracy?

If you are basing your need to do this on CAPM, MPT, and the Brinson, Hood & Beebower study, you have made a fundamental error in your judgment. We believe a conclusion opposite from the current interpretation should be drawn from the Brinson study. Instead of concluding that we need to focus on the sub-classes of assets we have invented over the last decade (the study showed sub-classes like foreign, small cap and style classes plus timing and security selection, accounted for less than 10% of the return variance), we believe that we should instead conclude that there is little effect of style biases, market capitalization biases, and the other smaller bets that investors may make as evidenced by the high r-squared despite these bets. That is what the data showed.