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How Many Stocks Make a Diversified Portfolio?

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Abstract

We show that a well-diversified portfolio of randomly chosen stocks must include at least 30 stocks for a borrowing investor and 40 stocks for a lending investor. This contradicts the widely accepted notion that the benefits of diversification are virtually exhausted when a portfolio contains approximately 10 stocks. We also contrast our result with the levels of diversification found in studies of individuals' portfolios.

I. Introduction

How many stocks make a diversified portfolio? Evans and Archer [9] concluded that approximately ten stocks will do. They stated that their results "Raise doubts concerning the economic justification of increasing portfolio sizes beyond 10 or so securities" (p. 767). Evans and Archer's conclusion has been widely adopted and cited in many current textbooks, but is it correct? No. The primary purpose of this paper is to show that no less than 30 stocks are needed for a well-diversified portfolio. A secondary purpose is to compare this finding to the levels of diversification observed in studies of individual investors' portfolios.

II. Portfolios and Risk

The risk of a stock portfolio depends on the proportions of the individual stocks, their variances, and their covariances. A change in any of these variables will change the risk of the portfolio. Still, it is generally true that when stocks are randomly selected and combined in equal proportions into a portfolio, the risk of a portfolio declines as the number of different stocks in it increases. Evans and Archer observed that the risk reduction effect diminishes rapidly as the number of stocks increases. They concluded that the economic benefits of diversification are virtually exhausted when a portfolio contains ten or so stocks.

Evans and Archer's conclusion has been cited in many textbooks. For example, Francis ([10], p. 749) wrote:

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> [P]ortfolio managers should not become overzealous and spread their assets over too many assets. If 10 or 15 different assets are selected for the portfolio, the maximum benefits from naive diversification most likely have been attained. Further spreading of the portfolio's assets is *superfluous diversification* and should be avoided. [Emphasis in the original.]

Stevenson and Jennings ([22], pp. 532–533) wrote:

> The results of the Evans and Archer study indicate that a portfolio of approximately eight to sixteen randomly-selected stocks will closely resemble the market portfolio in terms of fluctuations in the rate of return. Other studies have shown similar results and an unusual consistency using different time periods, different groups of stocks, and different research techniques. Consequently, while the CAP model requires the purchase of the market portfolio, essentially the same result can be achieved from a practical standpoint with a much smaller portfolio.

Gup ([11], pp. 363–364) wrote:

> Proper diversification does not require investing in a large number of different industries or securities . . . [T]he diversifiable risk is reduced as the number of stocks increases from one to about eight or nine . . . [W]hen the number of securities is increased to about nine, almost all of the diversifiable risk is eliminated.

Reilly ([18], p. 101) wrote:

> In terms of overdiversification, several studies have shown that it is possible to derive most of the benefits of diversification with a portfolio consisting of from 12 to 18 stocks. To be adequately diversified does *not* require 200 stocks in a portfolio [Emphasis in the original.]

Early studies, including that by Evans and Archer, reached their conclusions by simulating the relationship between risk and the number of stocks. Elton and Gruber [7] investigated the relationship between risk and the number of stocks in a portfolio further and provided an analytical solution for the relationship between the two.[1] Elton and Gruber's results, presented in Table 1, imply that 51 percent of a portfolio standard deviation is eliminated as diversification increases from 1 to 10 securities. Adding 10 more securities eliminates an additional 5 percent of the standard deviation. Increasing the number of securities to 30 eliminates only an additional 2 percent of the standard deviation.

III. The Costs and Benefits of Diversification

The principle that marginal costs should be compared to marginal benefits in determining the optimal levels of production or consumption is fundamental to economic theory. The fact that "almost all" of the portfolio's unsystematic risk is eliminated when it contains 10 or 100 stocks is meaningless when presented by itself.

Diversification should be increased as long as the marginal benefits exceed the marginal costs. The benefits of diversification are in risk reduction. The costs are transaction costs. The usual argument for limited diversification is that marginal costs increase faster than marginal benefits as diversification increases. For

[1] Bird and Tippett [1] have shown that studies using the simulation methodology are deficient. In particular, simulation studies tend to exaggerate the rate of decline in portfolio risk as the number of stocks in the portfolio increases.

TABLE 1
Expected Standard Deviations of Annual Portfolio Returns

Number of Stocks in Portfolio	Expected Standard Deviation of Annual Portfolio Returns	Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.236	1.00
2	37.358	0.76
4	29.687	0.60
6	26.643	0.54
8	24.983	0.51
10	23.932	0.49
12	23.204	0.47
14	22.670	0.46
16	22.261	0.45
18	21.939	0.45
20	21.677	0.44
25	21.196	0.43
30	20.870	0.42
35	20.634	0.42
40	20.456	0.42
45	20.316	0.41
50	20.203	0.41
75	19.860	0.40
100	19.686	0.40
200	19.423	0.39
300	19.336	0.39
400	19.292	0.39
450	19.277	0.39
500	19.265	0.39
600	19.247	0.39
700	19.233	0.39
800	19.224	0.39
900	19.217	0.39
1000	19.211	0.39
Infinity	19.158	0.39

Source: Elton and Gruber [8], p. 35. Portfolios are equally weighted. Elton and Gruber reported variances of weekly returns. We have converted these to standard deviations of annual returns.

example, Mayshar [17] developed a model that shows that it is optimal to limit diversification in the presence of transaction costs.

Comparison of benefits and costs requires a common measure. We use returns as our measure. The risk reduction benefits of diversification, in units of expected return, can be determined through a simple comparison of any two portfolios. The analysis is similar to that by Blume and Friend ([5], pp. 52–58).

We use a 500-stock portfolio as our benchmark portfolio and compare other, less diversified portfolios to it. We use a 500-stock portfolio as an example of an attainable, fairly diversified portfolio, but we claim neither that a 500-stock portfolio is a proxy for the market portfolio, nor that we cannot obtain better diversified portfolios.

The 500-stock portfolio can be levered, through borrowing or lending, to form portfolios $P(n)$ with combinations of expected returns and standard deviations according to the equation

$$(1) \qquad E\left[R_{P(n)}\right] = \left(R_F+\alpha\right) + \left\{\frac{E\left[R_{P(500)}\right] - \left(R_F+\alpha\right)}{\sigma_{P(500)}}\right\}\sigma_{P(n)},$$

where $E[R_{P(n)}]$ = the expected return of portfolio $P(n)$,
R_F = the risk-free rate,
α = the excess of the borrowing rate over the lending or risk-free rate for a borrowing investor, and zero for a lending investor,
$E[R_{P(500)}]$ = the expected return of the 500-stock portfolio,
$\sigma_{P(n)}$ = the standard deviation of portfolio $P(n)$, and
$\sigma_{P(500)}$ = the standard deviation of the 500-stock portfolio.

Equation (1) defines what we will call the 500-stock line and all portfolios $P(n)$ lie on it (see Figure 1). The 500-stock line is composed of two segments. The first, from R_F to $P(500)$, represents the portfolio combinations for a lending investor. The lending rate is R_F, the risk-free rate. The second, from $P(500)$ through $P(10)$, represents the portfolio combination for a borrowing investor. The borrowing rate is $R_F+\alpha$, where α is the excess of the borrowing rate over the lending rate.

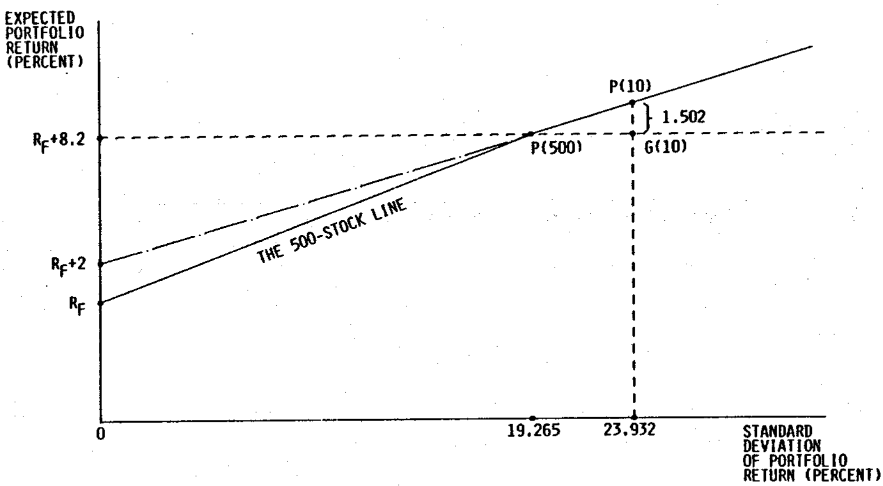

FIGURE 1*

* An expected return 1.502 percent higher than that of a 500-stock portfolio, $P(500)$, is necessary to offset the higher risk, due to limited diversification, of the 10-stock portfolio $G(10)$. A portfolio $P(10)$ can be constructed by leveraging $P(500)$ where the expected return of $P(10)$ is 1.502 percent higher than that of $G(10)$, while the two have identical risks. Data on standard deviations of portfolio returns are from Table 1. An estimate of 2 percent was used for α, the excess of the borrowing rate over the lending rate. An estimate of 8.2 percent was used for the risk premium.

Markowitz [16] developed a formula for the expected variance of a portfolio on n securities. That formula has been used by Elton and Gruber. We assume, as in Elton and Gruber, that an investor draws randomly from all stocks to form portfolios that differ in the number of stocks but have identical expected returns. We use the findings of Ibbotson Associates [12] about the risk premium on a particular 500-stock portfolio, the Standard and Poor's (S&P) 500 Index. Note that the S&P 500 Index is one attainable 500-stock portfolio. While Elton and Gruber use equally weighted portfolios, the S&P 500 Index is value weighted. We assume, for now, that the cost of maintaining an equally weighted 500-stock portfolio is identical to the cost of maintaining a value weighted 500-stock portfolio.

We use an Ibbotson Associates ([12], p. 42) estimate of the risk premium on the 500-stock portfolio, $E[R_{P(500)}]-R_F$. The arithmetic mean of the risk premium over the period 1926–1984 is 8.2 percent per year. We use 2.0 percent per year as an estimate of α, the excess of the borrowing rate over the risk-free or lending rate. The estimate is based on a comparison between the Treasury Bill rate, a proxy for the lending rate, and the Call Money rate. The Call Money rate is the rate charged on loans to brokers on stock exchange collateral (i.e, margin loans), and it provides a starting point for the estimate of the borrowing rate. The Call Money rate is less than 2 percent higher than the Treasury Bill rate (see Table 2). However, brokers typically charge their borrowing customers somewhat more than the Call Money rate. An estimate of 2 percent for α seems reasonable.

TABLE 2
Difference between the Call Money Rate and the Treasury Bill Rate

Date	(1) Call Money Rate (percent)	(2) Treasury Bill Rate (percent)	(1)−(2) Difference
Jan. 15, 1985	9.25	7.74	1.50
Feb. 15, 1985	9.38	8.20	1.18
March 15, 1985	9.75	8.48	1.27
April 15, 1985	9.50	8.14	1.36
May 15, 1985	9.00	7.69	1.31
June 14, 1985	8.63	7.21	1.42
July 15, 1985	8.63	6.92	1.71
Aug. 15, 1985	9.25	7.14	2.11
Sept. 15, 1985	8.75	7.22	1.53
Oct. 17, 1985	8.88	7.20	1.68
Nov. 15, 1985	9.13	7.21	1.92
Dec. 16, 1985	9.00	7.05	1.95
			1.58

Source: Data are from the Money Rate tables of the *Wall Street Journal* on the specified dates. We used data from the *Wall Street Journal* for the 15th of each month during 1985, or a date close to the 15th if data for the 15th were not available. The Call Money rate is the mean of the range of rates provided. The Treasury Bill rate is the rate for the most recent auction of 13-week bills.

To calculate the risk reduction benefits of diversification, compare, for example, a portfolio of ten randomly selected stocks, $G(10)$, to a portfolio $P(10)$, that lies on the 500-stock line and has a standard deviation identical to that of portfolio $G(10)$. We know from Elton and Gruber (Table 1) that portfolio $G(10)$

has an expected standard deviation, $\sigma_{G(10)}$ of 23.932 percent, and that the expected standard deviation of the 500-stock portfolio, $\sigma_{P(500)}$, is 19.265 percent. The standard deviation, $\sigma_{G(10)}$, exceeds $\sigma_{P(500)}$ as portfolio $G(10)$ contains more diversifiable risk than portfolio $P(500)$.

If stocks are chosen randomly, every stock and every portfolio has an expected return of R_F + 8.2 percent, composed of the risk-free rate and an 8.2-percent risk premium. Thus, the expected returns of both the 10-stock portfolio, $G(10)$, and the 500-stock portfolio, $P(500)$, are equal to R_F + 8.2 percent.

How much would a portfolio $P(10)$, that levers the 500-stock portfolio, $P(500)$, be expected to yield if $P(500)$ were levered so that the standard deviation of the returns on portfolio $P(10)$ is 23.932 percent? Using Equation (1) we find that

$$E\left[R_{P(10)}\right] = \left(R_F + 2\right) + \left\{\frac{\left(R_F + 8.2\right) - \left(R_F + 2\right)}{19.265}\right\} 23.932 = R_F + 9.702 .$$

An investor obtains the R_F + 9.702-percent expected return on portfolio $P(10)$ by borrowing 0.242 of his or her wealth and investing 1.242 of his or her wealth in the 500-stock portfolio (see Figure 1).

The return differential between the levered 500-stock portfolio, $P(10)$, and the 10-stock portfolio, $G(10)$, is $E[R_{P(10)}] - E[R_{G(10)}] = [R_F + 9.702] - [R_F + 8.2] = 1.502$.

The 1.502-percent differential in the expected return between the levered 500-stock portfolio $P(10)$ and the 10-stock portfolio $G(10)$ can be interpreted as the benefit that an investor derives from increasing the number of stocks in the portfolio from 10 to 500. In general, the benefit from increasing the number of stocks in a portfolio from n to 500 is

$$\text{(2)} \quad E\left[R_{P(n)}\right] - E\left[R_{G(n)}\right] = \left\{\frac{\sigma_{P(n)}}{\sigma_{P(500)}} - 1\right\}\left\{E\left[R_{G(n)}\right] - \left(R_F + \alpha\right)\right\} .$$

For the 10-stock portfolio, discussed earlier, we have

$$E\left[R_{P(10)}\right] - E\left[R_{G(10)}\right] = \left\{\frac{23.932}{19.265} - 1\right\}\left\{\left(R_F + 8.2\right) - \left(R_F + 2\right)\right\} = 1.502.$$

Benefits, in terms of expected returns, of increasing the number of stocks in various portfolios to 500 are presented in Table 3.

We turn now from the measurement of the benefits of diversification to the measurement of its costs. Assume, for now, that no costs are incurred in buying, selling, and holding of portfolios $G(n)$ composed of less than 500 stocks. A leveraged 500-stock portfolio, $P(n)$, is preferable to a portfolio $G(n)$ if the costs of $P(n)$ are lower than the benefits that come with increased diversification.

A 500-stock portfolio is available to all investors in the form of the Vanguard Index Trust, a no-load index fund that mimics the S&P 500 Index. The fund provides a return that is lower than that of the S&P 500 Index because investors pay transaction costs and administrative expenses. The mean annual re-

TABLE 3

Difference between Expected Annual Return of a Portfolio of *n* Stocks, *G*(*n*), and Expected Annual Return of a Portfolio *P*(*n*) That Levers a 500-Stock Portfolio Such That Standard Deviations of Returns of Portfolios *G*(*n*) and *P*(*n*) Are Equal[a]

Number of Stocks in Portfolio (*n*)	Return Differences for Borrowing and Lending Investors: Borrowing Investor	Lending Investor
10	1.502	1.986
20	0.776	1.027
30	0.517	0.683
40	0.383	0.507
50	0.302	0.399
100	0.135	0.179

[a] The figures in this table were calculated using Equation 2 with data from Table 1. The risk premium is estimated as 8.2 percent, the arithmetic mean risk premium. Risk premium data are from Ibbotson Associates [12], p. 42. The value of α, the excess of the borrowing rate over the lending rate, was estimated as 2 percent.

turn differential for the years 1979–1984 is 0.49 percent (see Table 4). Of course, 0.49 is less than 1.502, so the Vanguard portfolio dominates a ten-stock portfolio even when the cost of buying, selling, and holding these ten securities is zero.

TABLE 4

Comparison of Returns to Investors in the Standard and Poor's (S&P) 500 Index and Vanguard Index Trust, 1979–1984

Year	(1) Rate of Return on S&P 500 Index (percent)	(2) Rate of Return on Vanguard Index Trust (percent)	Difference (1)−(2)
1979	18.44	18.04	0.40
1980	32.42	31.92	0.50
1981	−4.91	−5.21	0.30
1982	21.41	20.98	0.43
1983	22.51	21.29	1.22
1984	6.27	6.21	0.06
		Mean	0.49

Source: Vanguard Index Trust returns data are from Wiesenberger Financial Services [23]. S&P 500 Index returns data are from Ibbotson Associates [12].

Note that the Vanguard Index Trust serves only as an example of an attainable well-diversified and unmanaged mutual fund. Similar funds with various combinations of securities would be offered, if investors demand them.

A comparison of the 0.49-percent figure to the figures in Table 3 makes clear that the Vanguard Index Trust dominates a 30-stock portfolio, *G*(30), for a borrowing investor, and a 40-stock portfolio, *G*(40), for a lending investor, even if we assume that no costs exist for buying, selling, and holding stocks in portfolios *G*(30) and *G*(40) while the Vanguard Index Trust costs are paid.

The figures quoted above were obtained under a set of particular assumptions, and they may increase or decrease as the assumptions change. We will consider here some prominent cases.

First is the issue of transaction costs. So far we have assumed that investors pay the costs of the Vanguard Index Trust, but they pay nothing for buying and selling and holding stocks of less diversified portfolios, $G(n)$. This assumption leads to an underestimation of the advantage of the Vanguard Index Trust over portfolios $G(n)$. For example, consider the case where costs associated with portfolios $G(n)$ amount to 0.1 percent per year of the value of the portfolio. The effect of these costs on the relative positions of portfolios $G(n)$ and the Vanguard Index Trust is equal to the effect of reducing the Vanguard Index Trust annual costs by 0.1 percent, from 0.49 to 0.39. Such a change makes the Vanguard Index Trust superior to a portfolio of 35 stocks, rather than 30 stocks, for the case of a borrowing investor, and 50 stocks, rather than 40 stocks, for the case of a lending investor.

The estimation of annual costs associated with portfolios $G(n)$ is difficult because they depend on the interval between stock trades; costs are higher for those who trade frequently. However, the earlier example probably underestimates the advantages of the Vanguard Index Trust. The cost of a round trip stock trade is probably not lower than 1 percent, and the mean holding period of a stock is probably not much higher than one year. (See [20], p. 306.)

Second, the reliability of the standard deviation estimate for returns of portfolios consisting of few stocks is low relative to that of portfolios of many stocks. Elton and Gruber ([7], Table 8) reported that the standard deviation of the estimate of the standard deviation of the portfolio return is 1.8 percent for a portfolio of 10 stocks, and 0.3 percent for a portfolio of 50 stocks, but it drops to virtually zero for a portfolio of 500 stocks. We do not know how to measure the loss that is due to the inherent unreliability of the estimate of the standard deviation of portfolio returns in portfolios of few stocks. However, it is another disadvantage of low levels of diversification.

The case for the Vanguard Index Trust may have been overstated because of two reasons. First, investors may be able to choose superior stocks and use the returns on these stocks to compensate for the additional risk due to lack of diversification. Indeed, there is some evidence that investors are able to choose stocks that offer return advantages sufficient to eliminate some of the negative effects of transaction costs. For example, Schlarbaum, Lewellen, and Lease ([20], Table 14) found that individual investors had mean returns, after transaction costs, that were identical to the mean returns of mutual funds. However, Schlarbaum et al. adjusted only for the systematic risk of stocks in both individuals' portfolios and mutual funds. The lack of diversification in individuals' portfolios relative to that of mutual funds implies that individuals may do worse than mutual funds when proper consideration is given to both systematic and unsystematic risk.

Second, stocks in the Vanguard Index Trust are value weighted while the analysis here is based on equally weighted portfolios. It is possible that the cost of the Vanguard Index Trust underestimates the costs of an equally weighted 500-stock portfolio, since transaction costs per dollar investment are generally higher for small company stocks than for large company stocks.

IV. Do Individuals Follow Markowitz's Prescription on Diversification?

The framework in which individuals construct portfolios by choosing combinations of expected return and risk, measured as the standard deviation of the return, is a crucial building block for much work in finance. Markowitz developed the prescriptive (normative) framework.

An important prediction of the CAPM, a descriptive (positive) model based on Markowitz's idea, is that every investor would hold a portfolio of all securities available in the market (given efficient markets, perfectly divisible securities, and no transaction costs).

Evidence, however, suggests that the typical investor's stock portfolio contains only a small fraction of the available securities. Blume, Crockett, and Friend [3] found that in 1971, 34.1 percent of investors in their sample held only one dividend-paying stock, 50 percent held no more than 2 stocks, and only 10.7 percent held more than 10 stocks. A 1967 Federal Reserve Board Survey of Financial Characteristics of Consumers showed that the average number of stocks in the portfolio was 3.41 (see [4]). A survey of investors who held accounts with a major brokerage company revealed that the average number of stocks in a portfolio ranged from 9.4 to 12.1, depending on the demographic group [15].

Of course, the number of securities in the portfolio is not the sole determinant of the degree of diversification. Studies by Jacob [13] and others have shown that an investor can reduce unsystematic risk significantly with few securities if he or she chooses securities judiciously. However, there is no evidence that investors follow the suggested rules on optimal diversification with few securities. Blume and Friend ([5], p. 49) reported that the actual degree of diversification in 70 percent of the investors in their study *was lower* than suggested by the number of securities in the portfolios. Blume and Friend concluded that

> The empirical results show, however, that many investors, particularly those of limited means, do not hold well-diversified portfolios. The analysis of the returns realized by them confirms that these investors have exposed themselves to far greater risks than necessary (p. 58).

Observing individuals' stock portfolios provides only limited information about the level of diversification in their overall portfolios. While we know that there are only few stocks in the typical portfolio, it is possible that diversification is accomplished through bonds, real estate, and other assets. However, recent evidence by King and Leape [14] strongly suggests that limited diversification is observed even where assets other than stocks are included. Their study was based on a detailed survey of 6,010 U.S. households conducted in 1978. The survey oversampled high-income families and therefore provides a rich source of information on the composition of portfolios. One conclusion of King and Leape was that

> the differences in portfolio composition across households cannot be fully explained within the framework of the conventional portfolio choice model. The households in our sample, though wealthy, own a surprisingly small number of assets and liabilities, and this lack of diversification was found to be important when estimating asset demand equations. Given that the mean net worth of the sample was almost a quarter of a million dollars in 1978, it is hard to imagine that transactions costs, as traditionally defined, played a decisive role in producing incomplete portfolios (pp. 33–34).

It seems that a descriptive theory of portfolio construction, based on Markowitz, does not hold. People forego available opportunities for diversification, and transaction costs are not likely to provide a complete explanation for it.

V. Conclusion

We have shown that a well-diversified stock portfolio must include, at the very least, 30 stocks for a borrowing investor, and 40 stocks for a lending investor. This conclusion contradicts earlier results, quoted in many current textbooks, that the benefits of diversification for stock portfolios are exhausted when the number of stocks reaches 10 or 15. Moreover, observation of individuals' portfolios suggests that people do not hold portfolios that are well diversified.

Why do people forego the benefits of diversification? Maybe investors are simply ignorant about the benefits of diversification. If ignorance is the problem, education may be the solution. However, existing evidence does not warrant a claim that investors should indeed be educated to increase diversification.

Alternative approaches to portfolio construction exist. One is the framework in which investors are concerned about the skewness of the return distribution as well as with the mean and variance. (See, for example, [6].) The other is the "safety first" framework (see [19]). However, we are not sanguine about the ability of either of these two theories to provide an adequate description of the way portfolios are built because neither is consistent with the following two common observations.

First, people do not seem to treat their assets as parts in an integrated portfolio. For example, some people borrow at 15-percent interest to finance a car rather than "borrow" from the college education fund they have set for their young chidren that pays only 10 percent interest. As Black [2] wrote, people "keep their money in separate pockets." Second, people display risk seeking and risk aversion that varies with the various "pockets." Many people seek risk by buying lottery tickets, while they are extremely risk averse with assets in retirement accounts. (For a discussion of these issues in the context of portfolio construction, see [21].)

We have to know much more about investors' goals and preferences to develop a framework that describes how they form portfolios. Meanwhile, we should not rush to conclude that investors should be educated to hold fully diversified portfolios.

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